

**AN INVESTIGATION OF GRADE 11 LEARNERS' MATHEMATICAL
PREPAREDNESS IN A SELECTED NAMIBIAN SCHOOL
A CASE STUDY**

A thesis submitted in partial fulfilment of the requirement for the degree of

**MASTER OF EDUCATION
(Mathematics Education)**

of

RHODES UNIVERSITY

by

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March 2010

ABSTRACT

The proliferation in the number of schools offering junior secondary education in Namibia since independence in 1990 has led to an increase in the number of learners in the classroom and has created a wide range of mathematical proficiency among learners entering senior secondary education in grade 11. This broad range of basic mathematical ability among these learners, together with increasing classroom numbers has caused problems for the senior secondary mathematics teachers (Batchelor, 2004).

The study shows that diagnostic testing can prove to be useful in assessing learners' mathematical preparedness by identifying learners' areas of weakness, which have hindered their mathematics learning and performance. Taking the results of a diagnostic test into consideration could help teachers cater for their learners who need remediation classes as early as possible before extending the mathematics curriculum. Setting and using diagnostic testing requires careful consideration; there are many pitfalls that are highlighted in this research. These include question coverage and general analysis of category totals.

ACKNOWLEDGEMENTS

Almighty God has provided me with strength, good health and perseverance/endurance to complete this research. I have learned to count my blessings one by one.

My supervisor, Dr Bruce Brown, with his professional assistance and constructive comments and criticisms contributed hugely to my professional growth. There was a time when I wanted to throw in the towel, but his encouragement kept me going. I am ever indebted to him.

I am also thankful to my lovely and understanding husband, Iyeleni and our lovely children; Konawa, Tangeni, Mekeloye, Shaningika and Ndinomukumo for their patience and support during my study. My wonderful mother, Albertina, always inspired and encouraged me to “Dare to Dream”.

My very special thanks go to Oshana Education Region for permission to conduct this research as well as for rendering their services when I needed statistical data to support my arguments.

Lastly, I am grateful to my colleagues and friends who assisted in any way possible during this study. To PETROFUND for funding one year of my study I am forever thankful.

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ACRONYMS USED IN THIS STUDY

SMICT	Science, Mathematics and Information, Communication and Technology
STOA	Science and Technology Options Assessment
R	Ready
PR	Partial Ready
NR	Not Ready
PISA	Programme for International Student Assessment
JSC	Junior Secondary Certificate
NSSC/O/H	Namibia Senior Secondary Certificate/ Ordinary/ Higher
H/IGCSE	Higher/International General Certificate for Secondary Education
LCE	Learner-centred Education
RHS	Right- hand side
LHS	Left-hand side
TIMSS	Trends in International Mathematics and Science Study
SACMEQ	Southern Africa Consortium for Monitoring in Education Quality
MoE	Ministry of Education
MBESC	Ministry of Basic Education, Sport and Culture

CHAPTER ONE: INTRODUCTION

1.1 Introduction

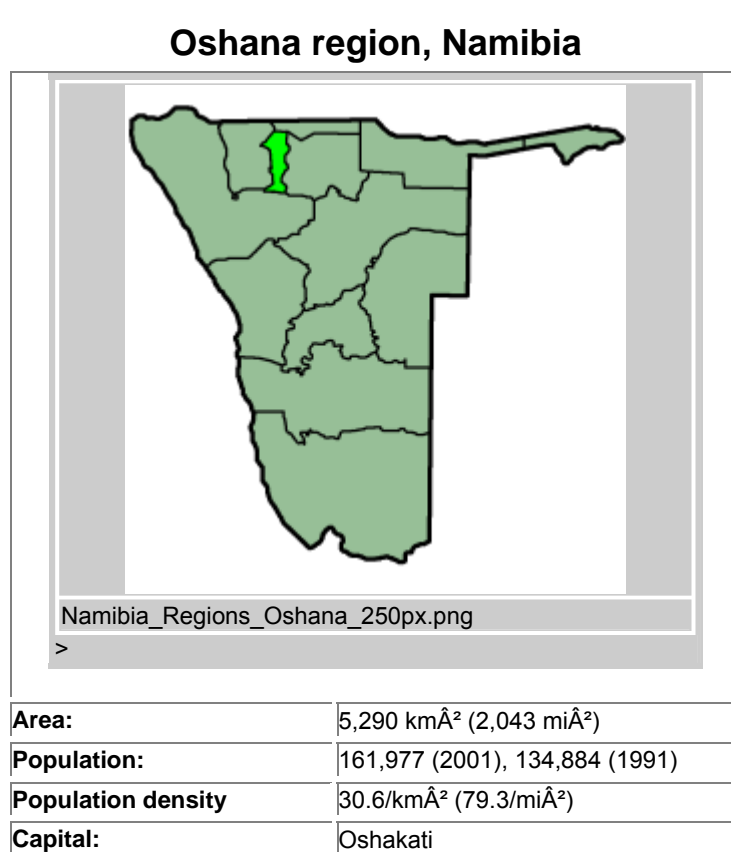
In this chapter I describe my research site, the context of my research (including my own personal experience) and research goals. Lastly, I provide an outline of the thesis.

1.2 Research site

Namibia is divided into thirteen political and educational regions. They are as follows: Oshana, Omusati, Oshikoto, Ohangwena, Karas, Khomas, Caprivi, Kavango, Otjozondjupa, Omaheke, Kunene, Erongo and Hardap. See Figure 1.2 below, for the demarcation of these regions. My research was conducted in the Oshana Region. The name Oshana lends itself well to this region as it describes the most prominent landscape feature in the area, namely the shallow, seasonally inundated depressions which underpin the local agro-ecological system. Although communication is hindered during the rainy season, the fish, which breed in the oshanas, provide an important source of dietary protein.

Figure 1.2 Oshana Region

(Source: Oshana at AllExpert
<http://en.allexperts.com/e/a/>)



The Oshana region has a population of 161 916 (2001 National Census report) of which 31% live in urban and 69% live in rural areas respectively. The Oshana Region has the second

highest population density in the country after Windhoek. The region consists of three fast growing towns in northern Namibia; namely Ongwediva, Oshakati and Ondangwa. The majority of businesses in northern Namibia are located here, providing a significant amount of employment. However, urbanisation is continuing and unemployment is at 41%. The region is linked to Tsumeb and other regions by a high quality trunk road and Oshakati and Ondangwa have airstrips. Reasonably good hospitals - both private and government are situated at Oshakati and Ongwediva, which also support a number of clinics. There is a reasonably good infrastructure, and most parts of the region are supplied with tap water and electricity.

The Oshana Region is a central point for education in the wider area, with two university campuses, a teachers' college, a teachers' resource centre, three libraries, and a vocational training centre. Although primary and secondary schools are spread across the region, they are insufficient. According to the statistics of Oshana Education Regional Office in 2009, there are 158 schools: 24 pre-primary schools, 65 primary schools of which 6 are private, 46 combined schools (from grade 1 to grade 10), 10 junior secondary schools, and 13 senior secondary schools. In 2009 there were 1 702 teachers and 53 041 learners in the region.

This study was carried out in one of the 13 senior secondary schools in Oshana Region. It is an urban school, having two Grade 8, two Grade 9, two Grade 10, nine Grade 11 and ten Grade 12 classes. The average class size comprises 38 learners. The learner enrolment for 2008 was 936. The school has 31 teachers and 37 institutional workers. It is also a boarding school.

1.3 Research context

The expansion of schools offering junior secondary education in Namibia since independence in 1990 and the subsequent increase in class size has created a disparity in the mathematical preparedness of learners entering senior secondary education in Grade 11. This wide variation in basic mathematical ability, coupled with increasing class sizes has led to various problems experienced by senior secondary teachers (Batchelor, 2004, pp. 69-74).

I have been a senior secondary mathematics teacher in this urban school from 1999 to 2009, teaching mathematics to Grades 11-12 for eleven years. Namibian mathematics for senior secondary is a two-year syllabus, and most of the Grade 11 and 12 mathematics teachers complain of the difficulty of completing this syllabus in two years. This arises because some

learners who enrol in Grade 11 have not acquired basic mathematical skills in Grade 10 and teachers find that they spend the first trimester of the school academic year revising Grade 10 mathematics.

In my teaching experience, this pattern has emerged over the years and is a matter of concern for most senior secondary mathematics teachers. Batchelor (2004) confirms that a lack of basic mathematical ability among learners has created problems for senior secondary mathematics teachers. Furthermore, recent studies (Vogel, 1997; Orhun, 2005; Britton, New, Sharma & Yardley, 2005; Lazarowitz & Lieb, 2006) show that most senior secondary learners around the world have below average basic mathematical competence.

This study was undertaken in order to diagnose learners' areas of weaknesses and thereby enable Grade 11 teachers to better understand and respond to the situation. I hope that the insights gained in this study will help teachers to improve their planning of teaching, by considering possible factors and recommendations emerging from this research.

This study will also contribute to the educational research field by providing insights into the mathematical preparedness of Namibian learners in comparison with other countries. I also hope to provide an analysis of learners' problem solving strategies to provide the reader with some insights into Grade 11 learners' mathematical preparedness at the school. These insights are not generalizable to all Grade 11 learners' mathematical preparedness, as only a small sample of learners was used. A more comprehensive study needs to be carried out if generalizations are to be made.

Despite the modest scale and size of this case study, it has the potential to inform the Namibian Ministry of Education of the mathematical preparedness of some Grade 11 learners. It may also provide useful insights into the impact of in-service training of Junior Secondary teachers so that they become aware of the key topics where most learners have 'unstable knowledge', and which therefore require more emphasis in teaching.

1.4 Research goal

My key research goal is to investigate the possible use of diagnostic testing to provide some insight into learners' mathematical preparedness when entering senior secondary school in Grade 11 at a selected Namibian secondary school. This was done by designing a diagnostic test as a tool to assess mathematical preparedness and then analyzing learners' responses.

The expected conclusion would be about how successful the test was in predicting mathematical preparedness and therefore discussions about the strengths and possible problems with using diagnostic tests would ensue. I include insights into learners preparedness as interesting examples of what can be obtained from a test when it is properly used and discuss inferences that could not be made from the diagnostic tests – as examples of the shortcomings or limits of diagnostic testing.

1.5 Outline of the thesis

This study consists of five chapters, including this introductory chapter in which I have briefly described my research site, my research context as well as my research goals.

Chapter Two is a literature review. Here I explore definitions of mathematic preparedness, in both the Namibian context and elsewhere. I also outline the challenges and problems associated with the concept of mathematical preparedness and with diagnostic testing as a tool to determine learners' mathematical preparedness.

Chapter Three describes the research design and the methods used to collect the research data. I explain the reasons for choosing an interpretive paradigm using a case study method. I also describe the different research instruments such as diagnostic testing and interviews used to develop a case study of some Grade 11 learners' mathematical preparedness at a selected school. In this chapter, I also discuss the limitations of the methods used in this study.

In **Chapter Four**, I present and analyze the data obtained from both the diagnostic test and the interviews. In the analysis, I make a number of inferences about the participants' mathematical preparedness in different categories.

Chapter Five concludes the study by presenting the main findings and highlighting its limitations. I also provide a brief reflection on the research process; the lessons learnt and suggest possible areas for future investigation.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

This chapter begins by giving a general overview of the transition between educational phases, and then looks at challenges and problems associated with this transition. I then explore the idea of mathematical preparedness, its definition, both generally and with reference to the Namibian context. Next, I look at how one can investigate mathematical preparedness by assessing competencies. I then analyze various tools which can be used to investigate mathematical preparedness, specifically diagnostic tests and stimulated recall. This is followed by the discussion of a framework for analyzing strands of mathematical competence.

My focus is on the mathematics competencies expected at the start of the Namibian senior secondary phase. Hence my interest in this study is to investigate learners' mathematical preparedness upon entering Grade 11 and the subsequent diagnostic testing of basic mathematical skills.

2.2 The transition between educational phases

2.2.1 General views

Transitions are defined as “the movement from one state of certainty to another with a period of uncertainty in between” (Berliner, 1993, unpagged). From each level or phase of schooling to another, learners are expected to have acquired certain mathematical proficiencies or basic competencies that are interwoven and overlap, and in teaching, one needs to draw on them simultaneously (Kilpatrick, Swafford & Findell, 2001). Transitions between educational phases often result in a heterogeneous student population at the secondary level, creating problems of mixed ability teaching (Ottevanger, Van den Akker & De Feiter, 2006).

The transition from middle school to high school needs to be handled carefully if we are to ensure continuity of learning. According to Multimedia in Education (1997):

Where there is a close collaboration between a secondary school and feeder primary school, there is more likely to be a progression and continuity in all subject areas. In the current situation, primary school children as well as junior secondary learners have a wide variety of secondary school choices. This makes the liaison between the primary and secondary schools much more difficult. There is a need for co-operation between schools to overcome the risk of learners performing well below the national norms [unpagged].

Wylie & Hipkins (2006) note that “the transition from primary to secondary schools is getting increased attention from researchers as a context of concern about closing gaps in educational achievement and improving student engagement” (p.1). Catterall (1998, p. 28) also states that “ it is not just in the transition to secondary level that dips in learning can occur but also in earlier stages of schooling.” Therefore, lack of basic mathematical competence among senior secondary learners might have originated from elementary schooling; and be transmitted through the grades, growing out of proportion so that learners find themselves lacking vital mathematical competence (Kgobe, n.d.). Namibia. Office of the President (1999, p. 112) agrees with these research findings, highlighting the need to lay a strong mathematical foundation at primary level, if learners are to succeed at secondary level mathematics.

Carmody, Godfrey & Wood (2006, p. 97) argue that the key to effective transition lies in recognizing and building on children’s prior knowledge, understanding and achievement. They further argue that any transition, either in mathematics education or other discipline, always builds on students’ mathematical prior knowledge regardless of the nature of that prior knowledge. If the transition is not handled properly, then the transition will have a negative effect on learners.

2.2.2 Challenges and problems associated with the transition between educational phases

Transitions between educational phases in Namibia, in particular the transition from the junior secondary phase to senior secondary phase, have been exacerbated by the challenge of a wide variation in basic mathematical ability among learners who are entering the senior secondary phase. A lack of basic mathematical ability among learners has created problems for senior secondary mathematics teachers (Batchelor, 2004).

Effects related to the transition to secondary level have been noted in systems with different schooling structures, and at different ages (Wylie & Hipkins, 2006). Johnston-Wilder (1999) reveals that:

If transition from primary schools or middle schools to secondary or upper schools is not managed well, then pupils can lose up to a year of useful progress in mathematics. (p. 103)

For example, in New Zealand and in England, attention has been paid to make the transition process easier through defining ‘feeder’ schools for each secondary school (Wylie & Hipkins,

2006). Feeder schools are primary schools that provide secondary schools with prospective learners and there is a close working relationship between these schools to ease the transition.

Recent studies (Multimedia Education, 1997; Burgess, 2002; Berliner, 1993) have revealed that in order to ensure continuity of learning during this transitional period between educational phases, ways of reducing the impact of transfer on learners need to be developed. Lazarowitz & Lieb (2006) feel that one way of reducing the impact of transfer between educational phases is to develop a diagnostic test. The test would enable teachers to identify learners' difficulties in mathematics and to plan accordingly, so that they execute their lessons to the benefit of all learners. Johnston-Wilder (1999) shares this view with Lazarowitz & Lieb (2006), that learner formative assessment has to be carried out prior to teaching "to enable educators to establish learners' mathematical pre-knowledge and to plan effective teaching strategies" (p. 741) to the benefit of all learners.

Johnston-Wilder (1999) further emphasizes the importance of assessing what learners already know in mathematics "as acquired misconceptions can have serious implications for these pupils' ability to comprehend new topics" (p. 105). Wood (1998, p. 52) writes that if teachers establish learners' prior knowledge and use this in planning schemes of work, it would minimize the gap created by mixed ability among learners in the same grade. If teachers have planned ahead, compensatory classes could be offered as early as possible, thus bringing learners up to the same level and ready for the extension of the curriculum.

Another challenge emerging from the research is the issue of learners moving to the next grade without acquiring the necessary basic competencies in certain promotional subjects including mathematics. For example, Goulding (1999) stated that when learners move between educational phases they "have restricted or incomplete views of mathematical entities like fractions" (p. 40).

Recent studies (Vogel, 1997; Orhun, 2005; Britton, New, Sharma & Yardley, 2005; Lazarowitz & Lieb, 2006) have shown that most senior secondary learners around the world function at below the average of expected basic mathematical competencies. This problem persists through to their tertiary education, as Britton, et al. (2005) highlight that "lecturers

complain that students either do not have sufficient mathematics or are unable to apply it in context.” (p. 2)

Kgobe (n.d.) articulates this point:

Though there is a high gross enrolment rate in South African basic education, the numbers begin to drop quite dramatically in secondary school and achievement levels are alarmingly low so that learners are moving through the grades but without necessarily attaining the learning outcomes prescribed by the curriculum. (p.1)

This scenario is not unique to South Africa, as Namibia is experiencing the same problems. Statistics released by the Namibia. Ministry of Education [MoE] (2007) confirmed that the net enrolment ratio for grade 1 to 9 is far higher than the one in senior secondary. It further states that:

Most children leave school without the foundation skills and competencies they ought to have acquired. For example, functional literacy rates are low among grade 6 and even grade 10 graduates. Namibia was ranked the lowest of any country in the SAQMEC II test in mathematics and English reading at the primary level. Poor learning at primary level carries over to higher levels, especially in mathematics and science. (p. 18)

Habash, Suurtaam, Yagoub, Kara & Ibrahim (2006) share this sentiment and say “There is a gap in knowledge and skills expected among learners in a secondary school, that is more pronounced in mathematics” (p. 101).

Therefore, there is a need to tackle the problem of mixed knowledge and mixed ability intake in mathematics at all educational levels (Jane-White, 2000). If learners’ basic mathematical competencies, which are supposed to be attained from the lower primary and junior secondary curriculum, are low, then it will be difficult for learners to grasp the extension of mathematical knowledge required for the senior secondary phase (Kgobe, n.d. and Namibia. MoE, 2007). Furthermore, Chinnappan, Dinham, Herrington & Scott (2008) emphasised that if secondary learners are to achieve well in mathematics they should receive sufficient ‘grounding’ in Mathematics in primary school. A solid foundation of mathematics at primary to junior secondary phases will pave the way for learners as they proceed to the next level. Therefore, learners need to display at least basic mathematical competence if they are to comprehend the mathematics curriculum successfully.

It is an established fact that some, if not most, learners in secondary schools encounter learning difficulties in mathematics, and the learning process could be facilitated and enhanced “if such difficulties are known to the instructors and addressed in their teaching” (Lazarowitz & Lieb, 2006, p. 741).

Although some of the recent studies (Wylie & Hipkins, 2006; Orhun, 2005) have focused on learners’ mathematics preparedness and its relation to gender and socio-economic background, this study approaches the issue from the perspective of mathematical content knowledge of senior secondary learners with respect to the mathematics curriculum of the Namibian junior secondary phase.

Kilpatrick, et al. (2001) also highlights recent research findings on the “widespread dissatisfaction with student learning of mathematics in secondary school and beyond” (p. 256). Tanner & Jones (2000) share this sentiment “You may be surprised and disappointed at how many children have failed to grasp basic arithmetical concepts by year 8” (p. 147). These researchers emphasize the fact that learners have a poor foundation in mathematics at the senior secondary school level, a situation that also applies to Namibia.

2.3 Mathematical preparedness

2.3.1 Definitions of mathematical preparedness

According to Neufeldt & Guralnik (1994) preparedness is “the state of being prepared.” The word ‘prepare’ means “to get ready or to equip or furnish with necessary provisions accessories” or in other words to make one ready (p. 1064). Longman’s Active Study Dictionary (2005) further defines preparedness as “being ready or able to do something”. It also defines the word prepare as “to give someone the training or skills that they need to do something”. Oxford’s Advanced Learner’s Dictionary (1995) defines preparedness as “the state of being prepared, ready or willing”. The word prepare is defined as “to get ready or make something / somebody ready” for a certain task or purpose.

I would define mathematical preparedness as a state of being equipped with the necessary skills, competence, and knowledge to carry out mathematical processes proficiently, such as conceptual understanding, and being fluent in mathematical procedures. Blomhøj & Jensen (2003) define mathematical preparedness as “someone’s insightful readiness to act in a way

that is mathematically satisfying” (p. 124). When one is mathematically prepared, one should have some level of mathematical learning or functioning. A person should also use mathematical knowledge and skills in solving and interpreting problems efficiently as well as applying mathematical concepts in context.

Learners become prepared mathematically from an early stage, firstly through interaction with the family, at nursery school and later through formal education, from primary phase to secondary phase and beyond (Parker, 2005, p. 24). It is expected that at each level, learners should be in possession of sufficient mathematical knowledge and skills and that these are ready to be scaffolded (Kilpatrick et al., 2001, p. 57).

2.3.2 Mathematical preparedness in the Namibian context

In general, the formal education system in Namibia is divided into three main phases, some of which have combined phases: Pre-Primary Phase, Primary Phase (Lower Primary: Grade 1 - 4 and Upper Primary: Grade 5 - 7) and Secondary Phase (Junior Secondary: Grade 8 - 10 and Senior Secondary: Grade 11 - 12). Very few combined schools offer both primary and secondary phases. Learners at the end of the junior secondary phase (Grade 10) write a Junior Secondary Certificate (JSC) examination to qualify for the senior secondary phase (SACMEQ II Report, 2004, pp. 3-4).

The Namibian education system has adapted the senior secondary school “International General Certificate of Secondary Education” and “Higher International General Certificate of Secondary Certificate” (IGCSE/HIGCSE) examinations for the “Namibia Senior Secondary Certificate Ordinary Level” and “Namibia Senior Secondary Certificate Higher Level” (NSSC) examinations at the end of Grade 12 (Namibia. MoE, 2007).

The Namibian education system is based on the principles and practice of the learner-centred education (LCE) paradigm, which in return informs the Pilot Curriculum Guide for Formal Basic Education (Namibia. Ministry of Basic Education, Sport and Culture, 1996, p. 23). The Broad Curriculum then serves as a guideline for educators to achieve basic educational goals and aims. One of the key goals for Basic Education in Namibia is to promote “the development of functional numeracy and mathematical thinking” among learners.

Furthermore, Basic Education aims to “develop positive attitudes towards mathematics; enable learners to acquire basic number concepts and numerical notation; enable learners to understand basic mathematical concepts and operations; and enable learners to apply mathematics in everyday life” (Namibia. MBESC, 1996, p. 5). For the realization of these goals and aims, learners need to be equipped with a strong foundation in mathematics and the LCE approach should be enforced (Namibia. MBESC, 1996).

Namibia, MoE (2007) ensures that each subject, including mathematics, in the different educational phases, has a guide or syllabus that prescribes skills to be taught in order to prepare the learner for the next grade. Each learner is expected to have acquired basic competencies before proceeding to the next level. Competencies play a central role in the Namibian Pilot Curriculum Guide for Formal Basic Education because these describe what a learner should be able to do at the outcome of teaching and learning in each educational phase (Namibia. MBESC, 1996, p. 9).

2.3.3 Investigating mathematical preparedness

For the purposes of this study, I categorize ‘mathematic preparedness’ and design a diagnostic instrument according to the following basic competencies as stipulated in the Namibian mathematics curriculum for junior secondary phase (Namibia. MoE, 2006):

- Numbers and number relationships (Kilpatrick et al, 2001, pp.158-159)
- Measures (Kilpatrick et al, 2001, p.172)
- Algebra
- Space and shape
- Data handling and representations

Cohen, Manion & Morrison., (2000) state that “the purposes, objectives and content of the test will be deliberately fitted to the specific needs of the researcher in a specific, given context” (p. 322).

2.3.3.1 Mathematical competencies

According to the MBEC (1996), mathematical competence is defined as “what a learner should be able to do as the outcome of teaching and learning” (p. 9). PISA (2003), defined mathematical competence as “the fundamental mathematical knowledge and skills or an

ability to mathematise” (p. 27). Kilpatrick, et al. (2001, p.10) further emphasized that mathematical competence should reflect what learners know and are able to do.

I would define mathematical competence as the ability or knowledge to use basic mathematical skills to apply mathematical concepts in context and to solve mathematical problems in a variety of situations; in other words a mental activity from which mathematical understanding emerges. I argue that such basic mathematical skills form a solid foundation for mathematics, as learners need to draw simultaneously on those skills as they proceed through different educational phases (PISA, 2003; Kilpatrick, et al. 2001).

In my own teaching experience, it is very worrying to find a learner in senior secondary school who cannot add fractions such as $\left(\frac{1}{3} + \frac{1}{2}\right)$. This fits with Goulding’s (1999) statement that when learners move between educational phases they “have restricted or incomplete views of mathematical entities like fractions” (p. 40). A learner at senior secondary level should at the very least be able to carry out basic mathematical operations on numbers, as each year a learner is in school, he/she is supposed to become increasingly proficient in mathematics (Kilpatrick, et al., 2001, p. 135).

In Namibia, the Junior Secondary Certificate (JSC) is used as a benchmark (standardized test) for learners to enter Grade 11. According to Namibia. MoE, (2006), the Mathematics syllabus for the Junior Secondary Phase requires every learner upon completion of this phase to be:

confident in working with the set of real numbers; uses the calculator where appropriate; applies the concepts of ratio and proportion in solving problems; uses algebraic symbols and techniques; generates and solves simple equations; interprets and draws graphs of linear functions; can deduce geometrical properties of objects; draws inferences from statistical data and representations; and applies mathematics to a variety of everyday problems. (p. 6)

However, it seems that learners entering Grade 11 still exhibit different levels of basic mathematical competencies. This situation triggers some of the questions that I will engage with. Do learners meet the basic competencies expected upon completion of each grade? How do we assess or know what learners know and build on their prior knowledge?

2.3.3.2 Tools for investigating mathematical preparedness

Each mathematics curriculum outlines basic competencies which need to be achieved by learners before they are promoted to the next grade. Mathematical preparedness can be investigated by assessing this mathematical competence. Although there is a standardized test (JSC) in the Namibian education system to measure learners' basic competencies in mathematics, because of the wide variation in competence there is a need to incorporate alternative methods of assessment that are able to effectively assess the range of learners' mathematical competencies.

A *diagnostic test* is one of the criterion-referenced assessment tools which can be used to discover learners' strengths, weaknesses and difficulties as well as expose specific areas of weakness or strength (Cohen, et al., 2005, pp. 321-322). In the development of a diagnostic test it is important to consider what the test aims to assess (Dixon, 2005, p. 49). Pilot-testing needs to be conducted, in order to provide necessary inputs on the quality of the test and its usefulness (Bachman & Palmer, 1996, p. 234).

Students' prior knowledge is the important factor for meaningful learning. "Check this knowledge and teach according to it", writes Ausubel (1968, p. 22). Watt (2005, p.21) states that students may arrive with many misconceptions and these should be tackled as potential learning difficulties with new subject matter. He further writes that in order to allow learners to construct their knowledge in a new topic, their pre-knowledge can be identified by a pre-test before teaching commences.

Lazarowitz & Lieb (2006, pp. 741-742) agree with Watt (2005) that the results of this pre-test could provide teachers with valuable information on prior knowledge, misconceptions and learning difficulties, and may serve to identify areas of potential learning difficulties that could arise. A diagnostic pre-test can be developed and be "used as a tool for formative and remedial teaching in the instructional and learning process" (Lazarowitz & Lieb, 2006, p. 743). Such a pre-test can facilitate the learning process and could attend to learning difficulties.

Rather than separating the students, it has been demonstrated that the information from the diagnostic test can be used to determine a starting point and to assess the requirement for additional, optional resources (Dixon, 2005, p. 57). The use of diagnostic tests to assess the knowledge and understanding of learners has been described in the literature (Armstrong &

Croft, 1999; Batchelor, 2004). A range of uses of test results have been identified, including use in the prediction of student knowledge from prior qualifications to assess their 'probable preparedness' and use in methods of delivery (Dixon, 2005).

The diagnostic test described in the present study could be used both to influence the delivery of the curriculum and to assess the level of student understanding. As indicated by Bezuidenhout (1998), it is important that teachers "increase their awareness of students' possible misconceptions and take them into account when developing suitable strategies for teaching" (p. 397). Dixon (2005) also describes how diagnostic tests could be used "to predict areas that require particular attention and to develop appropriate teaching, learning and assessment strategies" (p. 55). The use of the formative assessment test as an instrument to encompass instructional activities can be performed by the teachers and their students, in order to provide information which can be used in a remedial mode to modify teaching and learning procedures in which they are engaged (Black & William, 1998, pp. 7-8).

Lazarowitz & Lieb (2006) suggested that "in future research, it is desirable to investigate the use of the criterion-referenced test" (p. 754), (e.g. diagnostic test) validated in this study and find out whether students' academic achievement could be enhanced as a result of the remedial instruction. A diagnostic test at the beginning of the instructional process may serve as a method for encouraging learners' interest and motivation in the subject area. A diagnostic test should therefore not be used in isolation, but other tools can be used such as in-depth interviews and stimulated recall for triangulation purposes (Dixon, 2005, p. 58). According to Cohen, et al. (2005) triangulation is a powerful tool in qualitative research to "demonstrate concurrent validity" (p.112).

Interview and Stimulated Recall

According to Meade & McMeniman (1991), stimulated recall sessions, where the learner verbalizes his/her thoughts underlying his/her recent actions, indicate a concentration on 'pedagogical content knowledge'. Stimulated recall generates greater insights into the relationships of learners' prior knowledge and actions, and minimizes the possibility of lack of basic competence in mathematics (Parsons, Graham & Honess, 1983). Stimulated recall can encourage a commentary upon the learner's thought process at the time of the test (Calderhead, 1981, pp. 211-217).

2.4 Framework for analysing mathematics competence

Kilpatrick, et al. (2001) used five strands of mathematical proficiency in analyzing learners' mathematical competence (p. 5). I use these strands in my study as they serve as a yard stick for assessing mathematical proficiency, namely:

Conceptual understanding - comprehension of mathematical concepts, operations, and relations. Knowledge that has been learned with understanding lays the foundation for generating new knowledge and for working out solutions to unfamiliar problems. When learners have acquired conceptual understanding in mathematics, they make connections between concepts and procedures, and are therefore able to argue with reasons why certain facts are a result of others (Kilpatrick, et al., 2001, p. 119).

Procedural fluency - is a skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Procedural fluency enables learners to deepen their understanding of mathematics concepts and makes learning skills easier and less prone to errors in calculations as well as to forgetting (Kilpatrick, et al., 2001, p. 122). When skills are learned without understanding, they are learned as tiny pieces of knowledge which are harder to connect to any new topics. Then learning new topics becomes difficult as there is no linkage between the previously learned concepts and skills and the new topic.

Strategic competence - is an ability to formulate, represent, and solve mathematical problems. Learners develop procedural fluency as they apply strategic competence in order to select from effective procedures. The learners' ability to formulate a problem so that they can use mathematics to solve it depends on their flexibility (Kilpatrick, et al., 2001, pp. 124-129).

Adaptive reasoning - is the capacity for logical thought, reflection, explanation, and justification. Learners need to be able to justify and explain ideas in order to develop reasoning skills, and have a clear understanding of mathematical concepts and procedures (Kilpatrick, et al., 2001, p. 130).

Productive disposition - is a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. Learners' beliefs play a vital role in the attainment of their mathematical proficiency. Being mathematically

competent depends on one's outlook on mathematics. Learners, who have developed a positive attitude towards mathematics, are confident in their knowledge and ability. They believe that with appropriate effort and experience, they can learn (Kilpatrick, et al., 2001, pp. 131-133). Attitudes towards mathematics learning hamper most learners. Therefore, learners' beliefs or phobias about mathematics might hold the key to unlocking the door to their lack of basic mathematical skills and understandings (Burgess, 2002, p. 253).

2.5 Conclusion

Diagnostic tests and stimulated recall can be used to investigate mathematical competence as long as proper care is taken in designing the instrument. The results can equip educators with the necessary and vital information which makes the execution of their duties more manageable. In the next chapter I present the research methodology used in this study.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

In this chapter I describe and justify the research methods used in this study. I explain the interpretive design of the research and discuss the use of the case study approach. I also describe the sampling of the research participants and the data collection methods (interviews and learners' test solution strategies). Finally, I explain how I validate and analyse my data.

3.2 Research design

This study follows the interpretive paradigm because I wanted to investigate the learners' mathematical preparedness upon entry to the senior secondary phase. As I defined in Chapter Two, mathematical preparedness is a state of being equipped with the necessary skills, competence and knowledge to carry out with ease mathematical processes such as conceptual understanding and fluency in mathematical procedures. I investigate mathematical preparedness by assessing learners' mathematical competence through diagnostic testing.

The interpretive paradigm enables me to engage in synthesizing and critically analyzing the data collected and to draw inferences about some Grade 11 learners' abilities to mathematise (Cohen, et al., 2000; PISA, 2003). Kilpatrick, et al. (2001, p. 10) further emphasizes that mathematical competence should reflect what learners know and are able to do. The learners' test solution strategies provide evidence of this competence. As is consistent with the interpretive paradigm, both qualitative and quantitative methods were used (Merriam, 2001).

I used the case study method. Gillham (2000) defined a case study as the investigation of an individual or a group to find answers to a specific research question and by providing reasonable evidence collected, get the best possible answers to the research question. The case consists of five Grade 11 learners selected from all the Grade 11 classes, except for Grade 11 G, at a school in Oshana Region, in 2008. Grade 11G learners were omitted because I was their Mathematics teacher and worked with them on daily basis. This omission thus eliminated possible threats to the research data because of the teacher-learner relationship.

3.3 Sampling and timing issues

3.3.1 *Sample site*

The school is situated in one of the three towns in Oshana Region, Namibia. The school has a capacity of about 936 learners and 31 teachers including the principal and three heads of

department. The school caters for learners from Grade 8 to Grade 12. The average number of learners per class is 38. Each year the school enrolls about 380 Grade 11 learners from most regions in the country. These learners usually operate at different levels of basic mathematical competence.

I selected this school for convenience purposes; it was where I work and therefore was close to where I live.

3.3.2 Sample selection and timing of the test

I designed and piloted a diagnostic test and administered the diagnostic test to learners to identify their areas of weakness and strength in foundational mathematics. Therefore the test was administered to a cohort of about 350 Grade 11 learners, excluding my class (Grade 11G).

My initial aim was to evaluate learners' basic mathematical preparedness before it was influenced by Grade 11 teaching. However, I did not manage to conduct the diagnostic test at the beginning of the school year as planned because I was waiting for the Regional Director and the Permanent Secretary's permission to conduct the research. This took longer than expected, and by the time the permission was granted by both offices, learners were busy writing the end-of-the term tests. The diagnostic test was thus finally administered to the full group of 350 learners towards the end of the first school trimester.

The late testing has impacted on the research goal. The initial goal was to test learners' mathematical preparedness upon entering grade 11. Due to delayed permission, the initial goal could not be realized as some teaching had already occurred. However, the impact of this teaching on test results was not unduly great, because much of the Namibian grade 11 mathematics curriculum for the first trimester deals with numbers and number relations and this is just a small portion of the grade 10 mathematics curriculum.

In addition, even though some teaching had occurred, it is not to be expected that the effects of poor preparedness would be undone by one trimester of straightforward teaching without carefully designed and implemented remediation. That is, poor preparation means poor initial proficiency and this lack of proficiency is quite a stable characteristic (Chinnappan, Dinham, Herrington & Scott, 2008). For this reason, it was judged worthwhile to continue with the

research goal as the results would still show some relation to the learners' mathematical preparedness. That is, the diagnostic test may still turn out to be a useful tool.

As I did not want to transgress any research ethics, I did not interview learners after they wrote the test for the first time. So, at the start of the second trimester, I invited all the remaining learners to write a repeat test on a voluntary basis. This resulted in a self-selected sample of 40. I re-tested these 40 grade eleven learners with exactly the same test as the first one.

I interviewed all 40 of the learners who wrote the repeat test. The first 6 interviews are spoilt due to technical error and after the first 24 interviews, the rest are spoilt due to 'imitation of treatment', therefore these interviews are not used for analysis (Burns, 1997). Finally, for analysis, I selected five final participants randomly; who all happened to be girls though the school was for mixed genders, from the 18 interviews which were not spoilt. I chose to focus on a relatively narrow sample of five learners as I believe that this restricted focus provides me with best means of getting rich data relative to the scale and extent of my investigation. I spent about six months collecting the data.

3.4 Data collection

I employed two data-collection tools to collect the research data. These were interviews and diagnostic tests of five selected Grade 11 learners. The multiple sources used allowed for triangulation, which adds texture, depth, and multiple insights to an analysis and can enhance the validity or credibility of the results (Cohen, et al., 2000). Teachers' judgments about the Grade 11 learners' mathematical preparedness were also used to shed light on the analysis.

3.4.1 Diagnostic test

According to Cohen, et al., (2000), a diagnostic test is an in-depth test to discover particular strengths, weaknesses and difficulties that a student is experiencing. By 'in-depth' I mean that certain level of depth which is possible with a single test hence learners were asked to show all their working which could be analysed in detail, rather than just choosing a presumed correct multiple choice letter. I designed the diagnostic test to test the mathematics preparedness of Grade 11 learners. The test was designed for an average Grade 11 learner, such "that the purposes, objectives and content of the test will be deliberately fitted to the specific needs of the researcher in a specific, given context" (Cohen, et al., 2000).

A literature research was carried out to identify patterns of questioning. In particular, the SAQMEC Report II (2004), PISA (2003), and TIMSS Mathematical Items (1996) assessment frameworks were investigated. It emerged from these standardized tests that the multiple-choice method was popular; therefore I designed the diagnostic test in the same format but provided more space for the candidate's answers. The space was used by the learners to explain their answer choice; see Appendix A for the test format. The diagnostic test covered basic competencies (numbers and number relations, algebra, measures, shape and space; and data handling) which are stipulated in the Namibian mathematics syllabus for the Junior Secondary Phase.

Pilot-testing was considered necessary in order to provide relevant inputs on the quality of the test and its usefulness (Bachman & Palmer, 1996, pp. 234). Bachman & Palmer (1996) also emphasize the importance of early pre-testing for providing qualitative feedback. The diagnostic test was trialled twice, using two different groups for different reasons; one was a group of four Grade 11-12 mathematics teachers and another was three Grade 11 G learners.

Firstly, the test was piloted with Grade 11 and 12 mathematics teachers in the selected school. A group interview with the Grade 11-12 mathematics teachers was conducted and their responses and feedback were used to improve the quality of the test design. Group interviews do not have only practical and organizational advantages, but also include the potential for discussion to develop, thus yielding a wide range of responses (Cohen, et al., 2000). After the teachers' group interview, I incorporated their recommendations and made the necessary adjustments to the test design. Then the diagnostic test was ready for the second trial.

The revised test was trialed with three Grade 11G learners to whom I was teaching mathematics. The results of the second trial were used to determine the language difficulty, level of mathematical difficulty as well as the duration of the test. The test was taken individually. This small trial allowed me to do a pre-analysis of the data and to do final adjustments before the test was administered to the target group of Grade 11 entrants for 2008. Appendix B shows an example of the final test.

After the amendments were done to the test, the final draft of the test was administered to 350 Grade 11 learners. This was done towards the end of the first trimester of the school year

because of the delay in gaining permission to conduct the research. The tests were marked and marks were recorded. The marking of the diagnostic test was done using criterion-referenced assessment in order to evaluate whether learners had the basic mathematical competencies required prior to entering Grade 11 (Bertrand & Cebula, 1980).

Due to time constraints I could not conduct the in-depth interviews after the initial test because the learners had started with their end-of-term tests. For this reason, the same test was re-administered for the second time to forty of the Grade 11 learners at the beginning of the second trimester of the Namibian school calendar. The test scripts were marked and marks were recorded as with the first round.

In my analysis, I compared learners' test results and solution strategies for both tests in order to see if the same error patterns occurred for each individual. The comparison will allow the reader to have a better understanding of the learners' mathematical preparedness because similar error patterns could imply a consistent weakness or 'unstable knowledge' in a learner's mathematical preparedness. It also enabled me to gauge the effect of repeating the test.

3.4.2 Interviews

A number of different interviews were carried out in this project for different reasons. These interviews included a teachers' group interview, pilot interviews with three Grade 11G learners and then the in-depth interviews with the five selected participants.

First the teachers' group interview was conducted with four Grade 11 mathematics teachers at the selected school. The teachers gave their feedback on the quality of the test and about what areas needed improvement. I received some useful feedback from this interview, such as using the same font and increasing the spacing throughout the test paper to improve the test format. I was also advised to rephrase the instruction for question six in order to clarify the question. Teachers also identified the need to correct three questions (11, 12 and 15) as there were no correct answers from the options given.

Finally, the teachers mentioned that the following topics were omitted: making a certain letter the subject of the formula, prime numbers and limit of accuracy; and suggested that they

should be included in the test. After the group interview, I edited the first draft of the test and incorporated the changes suggested.

Next, the pilot interviews were conducted with the three Grade 11G learners. This feedback was used to make further adjustments to the test. It was also used to get a feeling on how the actual interview would be and to see what sort of data the interview would generate. The pilot interview enabled me to adjust my interview schedule as well as to make the final touches to the diagnostic test. In the same vein, the pilot interviews enabled me to practice and improve my interviewing techniques. I learned not to ask leading questions, to be patient and a better listener.

Lastly, the forty in-depth interviews were done soon after the learners finished the second test; see Appendix C for interview schedule. These interviews allowed me an opportunity to ask questions focusing on the specific kinds of responses they had given in the test (Bachman & Palmer, 1996, p. 244). Five in-depth interviews were then randomly selected for analysis for this thesis. The in-depth interviews shed some light on and deepened my understanding of these five Grade 11 learners' basic mathematical competence. I used a tape recorder to capture the interviews and then later I transcribed the selected five interviews. Appendix D shows the interview transcripts.

3.5 Topic coverage for the test

The structure of the test was a pragmatic design decision based on the fact that everything could not be tested in a simple diagnostic test of limited duration. From my teaching experience, grade 11 learners tended to display diverse mathematical preparedness mostly in the following topics: numbers and number relations, measures, and algebra. For this reason, most items of the test were based on these topics. I included a single question on the remaining topics of data handling and geometry; because most learners had displayed sufficient basic competencies.

3.6 Data analysis

The learners' answers were marked using a simple marking scheme with a scale of 3 marks maximum per question. When an attempt was made but the answer and method of computation were incorrect, the learners were awarded 0 marks. One mark was awarded if only the correct answer was chosen from the four options without showing any working. It

was stated in bold letters on the cover page of the test and also emphasized verbally before the test that learners need to show their working to get good marks. Two marks were awarded when the answer was wrong but the computation method was correct; whereas three marks were awarded when both the answer and the method of computation were correct.

The learners' responses were analyzed according to the stipulated basic competencies stated in Namibian mathematics syllabus for junior secondary phase. According to Namibia (MoE 2006), the Mathematics syllabus for Junior Secondary Phase requires every learner upon completion of this phase to be:

confident in working with the set of real numbers; uses the calculator where appropriate; applies the concepts of ratio and proportion in solving problems; uses algebraic symbols and techniques; generates and solves simple equations; interprets and draws graphs of linear functions; can deduce geometrical properties of objects; draws inferences from statistical data and representations; and applies mathematics to a variety of everyday problems (p. 6).

Learners' results and solution strategies were then analyzed according to the following five categories: numbers and number relations, measures, space and shape, algebra, as well as data handling and representation. Firstly, the results were analysed by category totals. That is, the analysis looked at the total marks obtained by each learner in each category. The total marks were used to compare and judge learners' mathematical readiness. It was found that this did not yield much insight into learners' mathematical preparedness. Therefore the solution strategies were then analyzed according to theme totals within the categories. The thematic analysis yielded detailed data which clearly highlighted areas of mathematical weakness for each individual learner, even though they may have been judged as mathematically prepared in that particular category according to the analysis of category totals.

Atomic analysis was used to analyze the in-depth interview data because learners were asked to describe their thinking process step by step in relation to their written responses. This analysis produced qualitative data for the deeper understanding of the mathematics preparedness for these five Grade 11 learners to a certain extent. The process of atomic analysis has two phases: "atomization of the solving process and comparative analysis" with

their written responses (Stehlikova, 1999, p. 286). Atomic analysis can be characterized as a method whose aim is to investigate the student's intellectual processes.

3.7 Research ethics

Permission to carry out the research was obtained from the permanent secretary of the Namibian Ministry of Education, the Education Regional director as well as the principal of the selected school, see Appendix E permission letters. Participants were informed of the purpose of the research and assured of the anonymity and confidentiality. Appendix F shows the consent form. They were also given a chance to withdraw from the study whenever they felt so. Fortunately nobody did withdraw.

3.8 Validity and limitations of the research

Validity is the process of collecting and analysing evidence to support the inferences based on the instrument used (Fraenkell & Wallen, 1996). According to Cohen, et al. (2000) "validity in qualitative data might be addressed through honesty, depth, richness and scope of the data achieved, the participants' approaches, the extent of triangulation and the disinterestedness or objectivity of the researcher" (p. 105). The teachers' group interviews and the trialing of the test design with three Grade 11G learners which were done in this study, provide triangulation of the research data and this also contributes to the validity of the test.

Generally, internal validity refers to the accuracy of a study's findings with regard to the study subjects. In this case, internal validity refers to the extent to which the design and conduct of the trial eliminated the possibility of bias in the research data (Burns, 1997, unpagged). The internal validity was assured by making extracts from the test answer sheets, the transcribed data and the documents involved in this study available for inspection by interested parties.

By the time the school started at the beginning of the year, I had not received permission from the relevant authority to conduct the research. Therefore the diagnostic test was administered toward the end of the first school trimester when learners had already covered some work from the Grade 11 curricula. The diagnostic test was then re-administered at the beginning of the second school trimester (after a three-week school holiday), so that interviews could be conducted. Although there was a threat to internal validity because of repeating the test, it was minimized as I had accounted for events that occurred during the cause of the study that might have an impact on the final outcome (Fraenkell & Wallen, 1996).

I have used both data sets for the first and second tests in my analysis, and as expected in a repeated test there was an improvement between the first and the second results of the tests. In fact, it is only in the interviews that I only used data for the second test as the idea was to do a stimulated recall immediately after the test was taken; hence the reference to the second test's written data. Lastly, the repeat test does allow me some insight into learners' possible potential for learning and improvement. This makes it possible for me to make a much stronger judgment about not using the test for placement.

The interviews did not generate as much data as I expected. Most research participants did not express themselves well, irrespective of the probing questions; hence less was shared and said. After the first twenty-four in-depth interviews, the participants started to give answers for the following question before I had even posed that question. Therefore I detected the "diffusion or imitation of the treatment" (Burns, 1997, unpaged) as the participants might have shared information and experiences about the interview while I was busy conducting the interviews - over which I had no control. Therefore due to the 'diffusion of the treatment' I decided not to use any further interviews after the first twenty-four.

I also felt that because I was the head of the Mathematics Department, this had influenced the learners' responses to some extent because they answered what they felt I wanted to know, but not necessarily what they meant. Due to the sample size and scope of this study, my discussion will be limited to these five learners only.

3.9 Conclusion

In this chapter I have justified the research methods used in this study. I explained the research design and the approach which were used in this study. I also described the sampling of the research participants, the data collection methods (interviews and learners' test solution strategies), and how I analyzed the data.

CHAPTER FOUR: DATA ANALYSIS AND DISCUSSION

4.1 Introduction

Firstly, I give a brief description of the learners' profiles and then discuss the questions that are covered in the test and the score boundaries setting. I will also provide an analysis of each participant's test results with respect to the five categories: numbers and number relations, measures, space and shape, algebra and then data handling and representation. I then analyse the learners' solution strategies both per category and per theme. This includes the analysis of the interviews and focuses on the identification of learners' possible areas of weaknesses in mathematics, as well as the themes that emerge from the learners' solution strategies.

Finally, I pull the threads together, by identifying the strengths and weaknesses of the designed diagnostic test by comparing competence verses test scores, looking at the decisions made based on this diagnostic test and explaining how those decisions were made. A discussion of the test question coverage and the score boundaries conclude this chapter.

4.2 Profiles of research participants

This section gives a profile of each research participant in tabular form as well as a description of their schooling narratives. The profiles enabled the researcher to understand the background of participants and shed light on their mathematical experience prior to and during their schooling.

The table 4.2 below presents the profiles for the research participants.

Table 4.2 Profile of five Grade 11 learners

NO.	NAME	AGE	REGION	GRADE 7 MATHEMATICS SYMBOL	GRADE 10 MATHEMATICS SYMBOL
1	Anna	17	Ongava	C	C
2	Aune	17	Omhandanda	C	D
3	Aino	18	Omhandanda	B	A
4	Alma	17	Omhandanda	D	B
5	Abby	17	Omhandanda	A	B

Anna is a 17 year old girl. During the interview, Anna was a bit shy but she gained confidence as she settled and became comfortable with the process. She started school at the age of seven. Her Grade 8 - 9 mathematics teacher was her inspiration, and assisted her in developing a positive attitude towards mathematics. Anna was taught mathematics in every grade throughout her schooling. During primary education, she felt mathematics was difficult until she was advised otherwise. She now has a positive outlook on mathematics and at the interview she expressed her desire to learn more. Thanks to her Grade 8-9 mathematics teacher, she now knows how to use the scientific calculator effectively and the importance of showing one's workings.

Aune is also a 17 year old girl. Aune is confident and has a vision, as she clearly stated that she was studying hard so that she could become a medical doctor. Like Anna, she started school at the age of seven. Aune has also done mathematics in every grade throughout her schooling. In her own words she described her experience as "mathematics was okay, and up to now I'm coping" She realizes that as one progresses to the upper grades, the level of difficulty also increases. She confirmed that her foundation in mathematics from the lower grades was shaky (see interview transcript, line 23 to line 24, Appendix E). But she still has a positive attitude towards mathematics. She appreciated the effort of one student teacher who assisted her in learning multiples while in Grade 10.

Aino is a girl of 18 years of age. Aino is confident though she did not talk much. Aino has liked mathematics since day one at school. Her Grade 10 mathematics teacher played a role in the way she views mathematics today, because she told her that "without mathematics nowadays you will not be admitted at any institution." When she did not understand what was taught she said that she favoured remedial classes over a re-test.

Alma is also a self-confident girl of 17 years of age. Alma has learned mathematics from Grade 1 up to her current grade. In Grade 5 she felt that mathematics was too difficult for her. She could not pinpoint what was difficult then, but she just hated the subject for no specific reasons. Her view of mathematics and a negative attitude towards mathematics shifted for the better with compliments from her Grade 8 mathematics teacher, whose teaching strategies and attitude toward learners inspired this particular learner. Since then, she has enjoyed mathematics. She believes that if learners are to perform well in mathematics, a lot of practice is needed.

Abby is a confident 17 year old girl. She expresses herself fluently. Abby has also studied mathematics since Grade 1 and has been enjoying mathematics throughout most of her grades. In her own word she said:

In mathematics, I enjoy the ability to kind of ... to think about it and then you get that answer. And I also enjoy how sometimes an equation can be challenging to you, you just face it and you are there, like am I really suppose to do this? I mean the x plus y , where I am going to use them and stuff. And I also enjoy that time when you really to think of yourself kind of do it and until the last point you get the correct answer. You feel proud of yourself that I can actually use my mind and get something useful.

(A₅: line12, appendix E)

In the past, she thought mathematics was hard and also believed that she could not do mathematics. Her brother inspired her to have a positive attitude toward mathematics. In her own words she stated: “And from there on, I learn that it is better to have positive thought about it and I will also have positive marks.” Though Abby exhibited a positive attitude toward mathematics, she doubted the practicality of some of the mathematical concepts and topics.

Abby felt that mathematics teachers should demand that learners explain how they worked out their homework as some learners copied from others’ work without really grasping the key concepts or procedures.

4.3 Coverage of the questions in the test

The test consisted of 23 items, which were grouped into the five categories: numbers and number relations, measures, space and shape, algebra, as well as data handling and representation. Each category consisted of themes, for example;

- Numbers and number relations - four themes constituted of 18 items
- Measures - two themes constituted of 4 items
- Space and shape - one theme constituted of 2 items
- Algebra - three themes constituted of 7 items
- Data handling and representations - one theme constituted of 2 items.

4.3.1 Numbers and number relations

In this category, learners were assessed on basic arithmetic such as standard form, rounding off, and multiples, numbers in general, addition, subtraction, multiplication and division. The other parts which were assessed were ratio and proportion, money and finance, as well as fractions. This category consisted of more than two thirds of the questions in the test because number sense paves the way for general mathematics. The questions were grouped into four themes as indicated in the table 4.3.1 below. The total marks of the questions per theme were added up to get the total marks for each theme. The overall mark for this category was 43 marks.

Table 4.3.1 Numbers and number relations

Theme	Question No.	Total marks
Learners can:		
-Compute four basic operations, showing adequate knowledge of numbers and multiples	7, 9, 10, 15, 17, 20, 21	16
-Round off correctly, order and use scientific notation, converting between rational number representations	1, 2, 3, 4, 22	10
-Add or divide fractions effectively	6, 12	6
-Work out ratio, proportion and finances	7, 8, 15, 20	11
Total		43

4.3.2 Measures

In this category learners were assessed on their knowledge of unit conversion, for example mass, time and length. They were also tested on the volume of simple geometric shapes. Table 4.3.2 shows the template used to assess learners' working. The total mark for this category was 9.

Table 4.3.2 Measures

Theme	Question No.	Total marks
Learners can:		
-Convert units correctly	2,8,20,	6
-Compute the volume of basic shapes, draw a diagram and label given parts, use the correct formula	16	3
Total		9

4.3.3 Space and shape

In this category there was only one theme which assessed learners on angle properties and trigonometrical knowledge. The question grouping is represented in Table 4.3.3 and the total mark was 3.

Table 4.3.3 Space and shape

Theme	Question No.	Total marks
Learners can:		
-work out the trigonometry ratio, display knowledge of angle properties	14, 17	3
Total		3

4.3.4 Algebra

In this section, learners were tested on various aspects of algebra such as: evaluate and simplify, factorise and solve simple linear equations, as well as transformation of formula. Table 4.3.4 below shows the template of the themes in this category and the total mark was 18.

Table 4.3.4 Algebra

Theme	Question No.	Total marks
Learner can:		
-Evaluate and simplify; factorise and solve simple linear equations	9, 10, 11, 12, 13	14
-represent mathematical statements as an equation, transform formulae	5, 23	4
Total		18

4.3.5 Data handling and representations

There was only one theme in this category which dealt with probability and linear graphs. In this category, learners were assessed on extracting specific information from a statement and using it to answer the question. Table 4.3.5 represents the template used in the analysis of the test results. The total mark for this category was 4.

Table 4.3.5 Data handling and representations

Theme	Question No.	Total marks
Learners can:		
-Work out simple probability, in the simplest form -Interpret the graph.	18, 19	4
Total		4

4.4 Setting test score boundaries

Mathematics teachers (who were teaching Grade 11) were asked to generally categorize the mathematical readiness of all Grade 11 learners. These mathematics teachers had taught Grade 11 learners since the beginning of the year and at the time of the request, it was expected that they would have some insight into how well learners were doing in mathematics. The teachers' judgements were carried out immediately after the learners took the initial test. The three categories used by the teachers were as follows: *not ready* [NR], *partially ready* [PR] and *ready* [R].

I decided to make use of the initial test scores in order to determine the boundaries of the three categories of mathematics readiness namely:

- *Not ready [NR]* – would refer to the learner who lacked basic mathematical skills and knowledge such as basic arithmetic, introductory algebra, basic shapes and space, measures and data handling and representations. It was judged that this learner would need introductory lessons, before the formal classes. Scores that fell in the range of $0 \leq m < 35$ were taken to indicate that the learner was NR.
- *Partially ready [PR]* – referred to the learner who showed some mathematical basic skills, but these were either inadequate or inappropriate and so needed to improve in order to reach the desired level. Learners judged as partially ready would need intermediate remedial classes before formal classes. The range for this category range was taken as $35 \leq m < 45$, it was judged that any learner whose marks fell within this range would need some remedial classes to cope with the Grade 11 mathematics curriculum.

- *Ready [R]* – referred to a learner who showed adequate basic mathematical skills, such that s/he was at the desired or expected level of mathematics. It was judged that the learner could extend into the Grade 11 mathematical curriculum with ease. A learner in this category could opt for the remedial classes for her/his area of weaknesses. Any learner whose initial test score fell in the range of $45 \leq m \leq 100$, was regarded as ready to deal with the Grade 11 mathematics curriculum.

These boundaries were tested by looking at the teachers' judgement whereby one participant was categorised as NR, but all others were either PR or R. As predicted by the teachers, the learners' scores for the initial test fell within range of the category boundaries. This was done for all 350 learners and these boundaries worked for most of 350 learners.

4.5 Analysis of learners' test scores

Learners wrote the same diagnostic test twice (refer to Chapter 3 for the process and see Appendix A for test scripts). The initial test was given at the end of April, when learners had covered some of the Grade 11 curriculum. The second test was taken at the beginning of June, when learners had covered more work for Grade 11. Teachers' judgements of learner readiness were made just after the learners sat for the initial test. As can be seen in table 4.3 and figure 4.3, all learners show a tremendous improvement in the second test.

This was not surprising because this was the second time they had written the test and it was expected that the effect of practice would result in an improvement in performance. Their exposure to further Grade 11 work was also expected to improve their performance. In this section, the learners' total scores and possible relationships to teachers' assessments of readiness will be discussed.

Figure 4.5 below gives a graphical representation of the five learners' test scores in both the initial and the second tests. Though the test scores did not give a distinct indication as to which topics in mathematics learners had difficulty in, they did give an overview of the learners' performance in the tests taken. There were clear distinctions between the scores of different learners and these were used to categorise the learners in the above-mentioned categories.

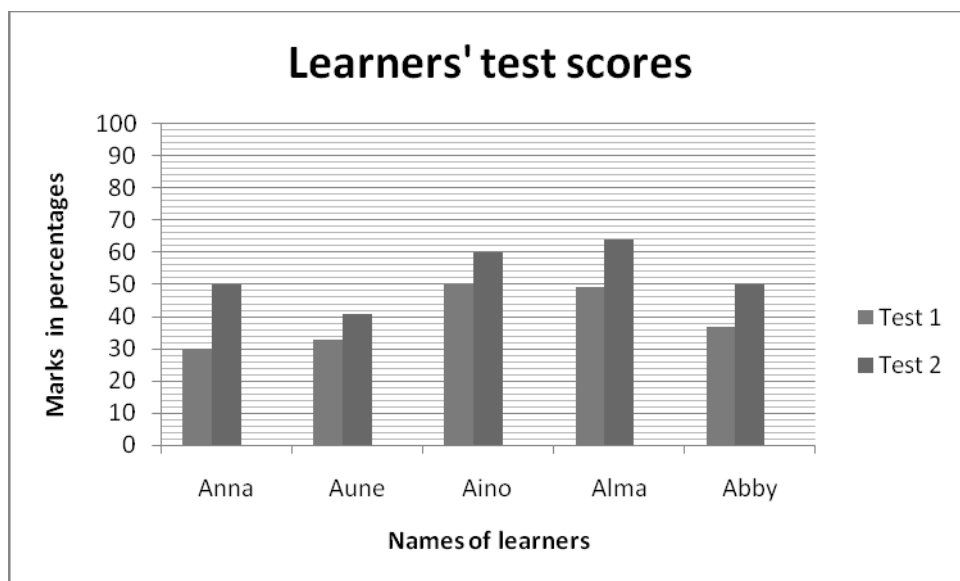


Figure 4.5 Comparison of learners' test scores in both of the tests

The table 4.5 below gives a summary of the learners' test scores in percentages for both tests as well as the teachers' assessments about the learners' mathematical readiness.

Table 4.5.1 Tests scores and teachers' assessments about learners' mathematical Preparedness

Name	Test 1 (%)	Test 2 (%)	Average % per learner	Teacher's judgement
Anna	30	50	40	Partially ready [PR]
Aune	33	41	37	Not ready [NR]
Aino	50	60	55	Partially ready [PR]
Alma	49	64	57	Ready [R]
Abby	37	50	44	Partially ready [PR]

Figure 4.5 above represents each learner's scores in both the initial test and second test. It is worth noting that the time between the tests was a three-week school holiday and hence there was no teaching and remediation taking place. In fact, the holiday may have had a negative impact on the performance of newly-learned content. *Aune* achieved the lowest average of the two tests - 37%, in comparison to other learners. Therefore Aune fell at the bottom of the sample as she got 33% in the initial test and probably can be referred to the NR category according to my early boundary criteria. Though her test scores increased from 33% in the initial test to 41% in the second test, her rate of improvement was the lowest when compared

to others in the sample. This indicates that there might be some weaknesses in her basic mathematical skills. Therefore it shows some correlation with the teacher's categorization of Aune' mathematical readiness as NR.

Although *Anna* got 30% in the initial test, her average scores for both of the tests was 40% which puts her in the same category as Abby whose average score was 44%. Anna showed a tremendous improvement after the initial test, compared to other participants. Possibly she did not settle down during the initial test or anxiety could perhaps have played a role in her initial test scores. According to the score boundaries in 4.3, Anna could have fallen into the NR category. But after the analysis of the second test I am in an agreement with Anna's teachers, who categorized Anna as a PR learner. Conducting an interview after the first test may have allowed Anna's potential to be more validly assessed. Anna's scenario was just one of the pitfalls in diagnostic testing. It was against this background that diagnostic tests should not be used for placement purposes as Anna could have been unfairly excluded.

Although Anna's scenario could be an isolated case, it emerged that when diagnostic testing is done, provision should always be made for the false positives and false negatives. Test scores alone should not be used to determine the learners' mathematical readiness as there could be many factors involved. Subsequent testing therefore needs to be done throughout the year in order to get a true reflection of learners' mathematical preparedness.

Abby got 37% in the initial test; therefore she fell into the PR category. She also improved her scores from the initial test to the second test. This is an indication that she learnt something during the period between the two tests. Abby's average score was 44% which indicates that the teachers' judgement about her was justified.

The other two learners, *Aino* and *Alma*, could be categorised as ready because their second test scores were in the sixty percentages. Both showed a great improvement from the initial test scores as compared to the scores of the second test. Their initial test scores were 49% and 50% respectively and they would have been classified as ready on the basis of this test as well. It emerged from their workings that they had a good foundation in basic mathematics in that they grasped the new mathematical concepts with ease and needed only to extend their mathematical prior knowledge. This supports the boundaries chosen to indicate learners considered being ready in mathematics and it also collaborates with the teachers' judgements to a greater extent.

Table 4.5.2 Learners' mathematical readiness verses teachers' judgements

Judgement	Total number of learners	
	Teachers	Test results
NR	50	63
PR	148	86
R	143	155

Note must be taken that about 37 learners did not take the test as they were absent during the test even though the teachers' judgement was done across the board. It emerged from this study that these boundaries were fit to be used for 'readiness testing' because they correlate well with the cohorts of learners' mathematical preparedness. Generally, the test results of most Grade eleven learners fell within the boundaries set for the test as well, therefore these test boundaries seem to be a basis of categorising learners according to their mathematical 'readiness'. The results from the test boundaries also confirm the teachers' judgements and this implies that 'readiness testing' coupled with teachers' judgements can be a useful technique to assess learners' mathematical preparedness.

4.6 Analysis of the learners' solutions strategies

As indicated in the previous section, the learners wrote the same diagnostic test twice. What was interesting was that though learners wrote the same test twice, and showed an overall improvement as expected, the same error patterns could still be detected in each learners' detailed solution attempts. Therefore it could be concluded that such individuals had difficulties in those particular areas. These error patterns and corresponding difficulties will be discussed in this section. The learners' writing, the scores for each question of the second test, and the data from the in-depth interviews will be used to analyse these patterns. My discussion will focus mainly on the scores of the second test because these scores informed the data of the in-depth interviews, which were conducted soon after all learners, had finished with the second test. In addition, selected examples of the second test would also be compared to the corresponding workings extracted from the initial test.

Finally, note that working from the second test which generally gave improved results, would give a more conservative assessment of learner weaknesses and this could improve the validity of the analysis of difficulties.

The test had 23 items, which were grouped in the five categories (see section 4.3) of:

- Numbers and number relations
- Measures
- Space and shape
- Algebra
- Data handling and representation

Though I have the five categories (which were divided into themes) above, less emphasis will be put on the two categories of space and shape, and data handling and representation, due to the fact that there was not enough coverage of these items in these particular categories. The remaining categories will be discussed in detail in the following paragraphs.

4.6.1 Learners' solution strategies per category totals

Figure 4.6.1 below shows a summary of each learner's performance per category. The performance for each category is described, by giving the marks attained as a percentage of the available marks for the category. The shorter bars in the graph indicate the topics in which the research participants lack the basic mathematical skills i.e. lower bars implies poorer performance. In a shape and space category, poor coverage also results in more discrete percentages (bigger jumps in the graphs), but the maximum is still 100.

The average of learners' scores per category in the last categories of algebra and data handling were very low compared to the first three categories. The percentages ranged from 0% to 44% respectively. It also emerged clearly from the learners' solution strategies that they had difficulties with the questions from the algebra and data handling categories. There was a good coverage of questions in the algebra category. Algebra counts for almost a third of the Namibian mathematics syllabus for Grade 11; therefore learners with this weakness might not successfully extend their learning to the Grade 11 curriculum.

The results show that the data handling and representations category needed to include more questions for better representation of the topic. Though the results were generally poor for data handling, there was not enough coverage of questions in the data handling category.

Therefore, it would be premature to draw any conclusions about learners' preparedness regarding this category.

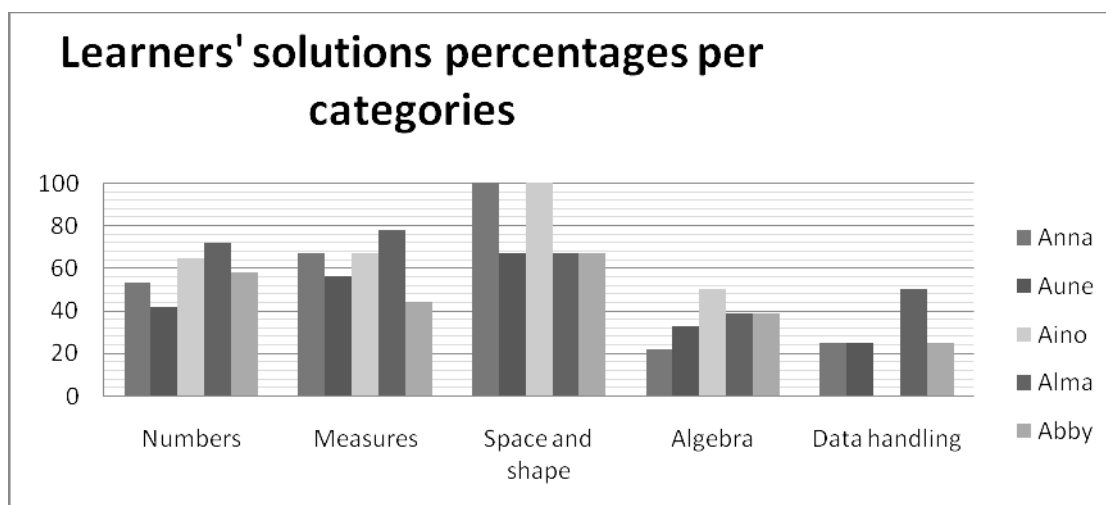


Figure 4.6.1 Learners' solution percentages per category.

The bar graph above indicates each learner's total scores per category, but as a total it does not give a clear representation of learners' mathematical preparedness. The percentage per category does not pinpoint specific areas where learners need assistance. Therefore, for a more detailed representation of learners' mathematical preparedness, an analysis of learners' solutions for each theme in every category is needed.

4.6.2 Learners' solution strategies per theme totals

The analysis of learners' solution strategies per theme created another dimension in the analysis of these diagnostic test results. The thematic analysis pinpoints the exact mathematical areas where these learners had difficulties, identifying learners' areas of weakness and hence their mathematical preparedness. In comparison to the analysis of learners' total marks per category, this analysis also gives a clear picture of which topics or subtopics learners might require assistance in. Instead of using percentages per theme, the actual marks a learner obtained per category were used; because these marks gave the actual representation of the learners' scores on a specific topic.

4.6.2.1 Numbers and number relations

Table 4.6.2.1 shows the template used in assessing individual learners' solution strategies for each theme in this category. The total marks per theme are given in the last column of the table. The overall marks for this category are given in the template as the total, which are 43.

Theme	Total marks
Learners can:	
1. compute four basic operations; show adequate knowledge of numbers and multiples;	16
2. Round off correctly, order and use scientific notations; converting between rational number representations	10
3. Add or divide fractions effectively	6
4. Work out ratio, proportion and finances	11
Total	43

Table 4.6.2.1 Numbers and number relations theme totals

Figure 4.6.2.1 gives a graphical representation of learners' scores per theme in this category. From the graph, it emerged that some learners had either difficulties or inappropriate mathematical basic knowledge for some of the themes.

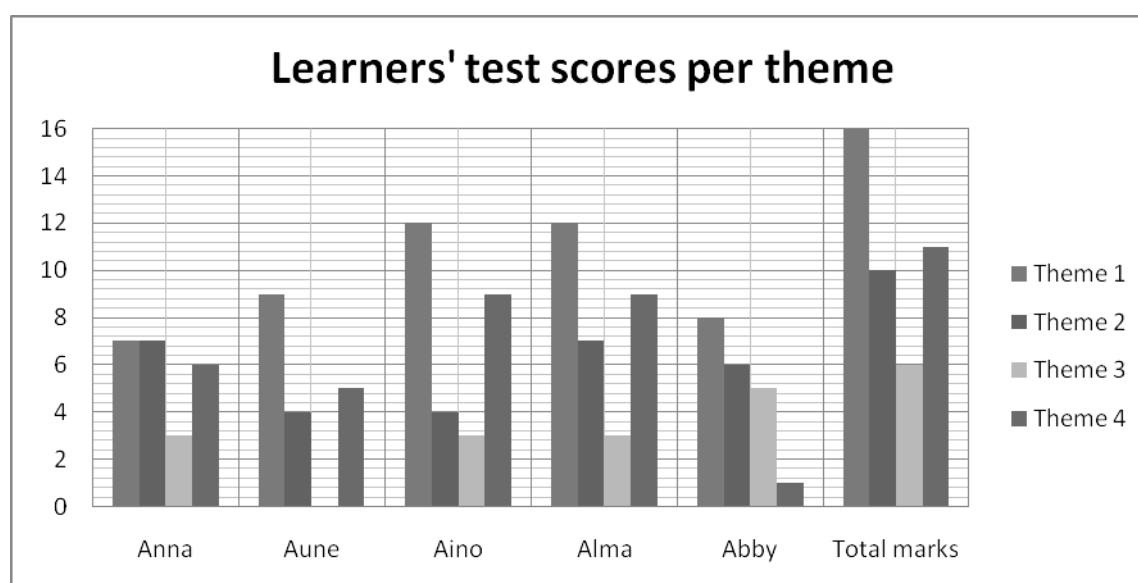


Figure 4.6.2.1 Learners' theme scores for the Numbers and number relations category

In the first theme, *Anna* scored the lowest marks compared to other participants. Although she displayed the key basic mathematical skills, she still needed remedial assistance in this theme. In the first theme a learner was expected to compute four operations and show adequate

knowledge of numbers and multiples. Let us take a glimpse of Anna's workings in examples A and B below.

Example A

Initial test

9. Evaluate $3xy - 2x^2$ when $x = -1$ and $y = 2$.

- A -4
- B -6
- C -8
- D 10

$$\begin{aligned}
 & 3x - 1 \times 2 - 2x^2 \\
 & = -6x - 2x^2 \\
 & = -4
 \end{aligned}$$

Second test

9. Evaluate $3xy - 2x^2$ when $x = -1$ and $y = 2$.

- A -4
- B -6
- C -8
- D 10

$$\begin{aligned}
 & 3x - 1 \times 2 - 2x^2 \\
 & 3x - 1 \times 2 - 2x - 1^2 \\
 & -6 - -2 \\
 & -4
 \end{aligned}$$

Though Anna could substitute the variables with a given value; she could not introduce brackets to distinguish between multiplying and subtraction in substitution of algebraic expressions. Instead of $(-1)^2$, Anna wrote -1^2 and she therefore failed to square -1, resulting in an error. In the initial test Anna did not show the substitution of any variable, but still got the same answer, -4. The fact that the same answer was obtained makes it seem reasonable that she had followed the same incorrect procedure as in the second test, without writing down a record of the substitutions. In any case, it seemed reasonable to assess that Anna did not display the skill of working with numbers and operations.

Example B

Initial test:

21. How many prime numbers are there between 30 and 40?

- A 0
- B 1
- C 2
- ☒ D 3

Second test:

21. How many prime numbers are there between 30 and 40?

- A 0
- B 1
- C 2
- ☒ D 3

31, 33, 37

X

From the workings above, I judged that Anna either did not know her multiples or she did not really understand the concept of prime numbers because she included number 33, as an example of a prime number. If she had known the multiples of 3 or 11 and understood prime numbers, she would not have concluded that 33 is a prime number. In the initial test Anna also concluded that there are 3 prime numbers between 30 and 40, but she did not list the three numbers she thought were the primes. It emerged clearly from her working above that her number sense had not improved that much since the initial test because she reached the same conclusion in both tests.

Aune got low marks in the second, the third and fourth themes respectively. It emerged from Aune's workings that she needed remedial classes in ratio, proportion and finances as well as rounding off, order and use of scientific notations as well as converting between rational number representations. She did poorly in most themes in this category. The graph also showed that Aune needed introductory lessons on fractions. Let us look at the following examples of Aune's workings:

Example C

Initial test:

5. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

A $\frac{2}{5}$

☒ B $\frac{2}{6}$

C $\frac{5}{6}$

D $\frac{1}{6}$

Handwritten mark: a small loop or flourish.

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} \checkmark$$

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} \times$$

Second test:

6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

☒ A $\frac{2}{5}$

B $\frac{2}{6}$

C $\frac{5}{6}$

D $\frac{1}{6}$

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{5} \times$$

Handwritten text: you added

In question 6, Aune's working indicated that her knowledge of fractions was clearly lacking. The addition of fractions was not done correctly in both tests. Aune's workings, both in the initial and the second tests, were quite similar with only a slight difference on the denominators. In the initial test, Aune had an idea of how to find the lowest common multiple of 2 and 3, but she then failed to work out the numerator. In the second test, Aune added the numerators even though the denominators were different, and then she added the denominators to get to her final answer. It was clear that her knowledge of fractions was shaky.

Example D

Initial test:

21. How many prime numbers are there between 30 and 40?

A 0

B 1

C 2

☒ D 3

30, 2, 5, 10

40, 2, 5, 10

30, 2, 5, 5

40, 2, 5, 10

Second test:

21. How many prime numbers are there between 30 and 40?

A 0

B 1

☒ C 2

D 3

Prime number are number that can divide a number
leaving a remainder, which are the same after ~~point~~

In the initial test Aune's answer was three. Her workings on this question proved also that she had no idea of what prime numbers were. In the second test, she gave a correct answer - 2, but it emerged from her explanation that she still did not grasp the meaning of prime numbers.

Aino did well in most themes in this category, especially in the first and fourth themes respectively. Although Aino was competent in the above-mentioned themes, she still had difficulties in dividing fractions, rounding off, order and use of scientific notations themes as well as converting between rational number representations.

Example E

Initial test:

3. Which one of the following numbers is the smallest

60% 0.67, $\sqrt{0.38}$ $\frac{2}{3}$

A $\frac{2}{3}$

B 60%

☒ C $\sqrt{0.38}$

D 0.67

$\sqrt{0.38}$ is smallest because if you simplify it you will get 0.62

Second test:

3. Which one of the following numbers is the smallest

60% 0.67, $\sqrt{0.38}$ $\frac{2}{3}$

A $\frac{2}{3}$

B 60%

☒ C $\sqrt{0.38}$

D 0.67

if you compare the square root of $\sqrt{0.38}$ with others item, you will actually the square one is smallest. e.g. 60% won't change, 0.67 if it's over 100 you will get $\frac{67}{100}$, while $\sqrt{0.38}$ you will get 0.616 if $\frac{2}{3}$ you will get 0.666 i.e. the third item got the least digit second digit after the comma

In both the initial and the second tests, Aino's conclusion was the same - $\sqrt{0.38}$. In both her explanations, Aino calculated correctly between her decimal fractions but she failed to convert 60% to decimal form. It is judged that she did not understand percentage and its relationship with fractions; hence she did not convert the percentage to a decimal fraction. Therefore this makes it difficult for her to compare the given numbers and choose the smallest.

Example F

Initial test:

12 Simplify

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

A $\frac{2x^3}{5}$

B $\frac{2x^3}{20y^3}$

C $\frac{10x^3}{50y^3}$

D $\frac{x}{5y}$ ✓

$$\frac{\cancel{1} \cancel{2} x^2}{\cancel{10} y^2} \div \frac{\cancel{5} x}{\cancel{5} y} = \frac{x}{5y} \quad \checkmark$$

2

Second test:

12 Simplify

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

A $\frac{2x^3}{5}$

~~B~~ $\frac{2x^3}{20y^3}$

~~C~~ $\frac{10x^3}{50y^3}$

D $\frac{x}{5y}$

multiply the numerator with the numerator and add the indices, as where as the denominator with another denominator, then the will ~~add~~ each other because the are all like at both side

✗

0

In the initial test, Aino did extremely well in this question. Her conclusion was also correct. But in the second test, she did not get it right and her explanation did not show her understanding of fraction knowledge. If Aino had a stable knowledge of fraction division, she would have been expected to get the correct answer in the second test; therefore she appeared to have a mixed and conflicting knowledge of fractions.

Alma did exceptionally well in the first, second and fourth themes respectively; she got half or more of the total marks per theme. In comparison with other participants, Alma displayed the necessary basic mathematical skills required in those themes (see figure 4.6.2.1). It emerged from the graph above that Alma also had difficulties with items in the third theme - adding or dividing fractions. Alma's inconsistency in working out items involving fractions, in both tests showed that her basic knowledge of fractions was very sketchy. Let us examine some of her workings:

Example G

Initial test:

6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

☒ A $\frac{2}{5}$

B $\frac{2}{6}$

C $\frac{5}{6}$

D $\frac{1}{6}$

because $1 + 1 = 2$ And $3 + 2 = 5$, so 2 becomes the numerator and 5 a denominator this is because when we are adding fractions Proper Fractions we add as directed.

Second test:

6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

A $\frac{2}{5}$

B $\frac{2}{6}$

☒ C $\frac{5}{6}$

D $\frac{1}{6}$

Find the LCD by multiplying 3×2
divide the LCD by 3 and by 2
add the product

$$\begin{array}{r} \frac{1}{3} + \frac{1}{2} \\ \frac{2+2}{6} \\ = \frac{4}{6} \end{array}$$

In the initial test, Alma did not demonstrate the basic skills and understandings of fractions and through her explanation one could clearly see that she had a vague knowledge of fractions. Her second test workings and explanation were all correct. By then they had

covered more of the Grade 11 curriculum, and in doing so, she had greatly improved her working with fractions.

Example H

Initial test:

12 Simplify

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

A $\frac{2x^3}{5}$

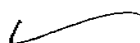
B $\frac{2x^3}{20y^3}$

C $\frac{10x^3}{50y^3}$

D $\frac{x}{5y}$

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

$$\frac{x}{5y} \rightarrow$$



Second test:

12 Simplify

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

A $\frac{2x^3}{5}$

B $\frac{2x^3}{20y^3}$

C $\frac{10x^3}{50y^3}$

D $\frac{x}{5y}$

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

$$2 \div 5 = 10$$

$$10 \div 5 = 50$$

$$x^2 \times x = x^3$$

$$y^2 \times y = y^3$$

$$\Rightarrow \frac{10x^3}{50y^3}$$



In the initial test, Alma did extremely well in this question and her conclusion was also correct. In contrast, Alma's working was off-track and the basic skills of fractions were not displayed in the second test. Here, Alma has made exactly the same change as Aino.

This is a good example of a negative result of teaching, whereby learners do not consistently implement a learned algorithm. Therefore it was judged that the contradictions in her solutions proved that she also has an unstable understanding of fraction division. Hence, she needs remedial classes in fractions in order for her to gain a proper understanding of fractions.

Abby had also displayed adequate basic mathematical skills in the first three themes, but she scored the lowest mark in the ratio, proportion and finance theme compared to other participants. Abby's knowledge on ratio and proportion was a bit shaky because she could not interpret the time correctly and also she did not display the understanding of ratio in her solution. Therefore Abby needs remedial lessons on time, ratio and proportions.

Example I

Initial test

8. It takes 4 hours for 2 men to dig a hole. How long would it take 6 men to dig the hole of the same size?

A 1h30min

B $1\frac{1}{3}h$

C $\frac{3}{4}h$

D 1h33min

$$= \frac{4 \text{ hours} \times 2 \text{ men}}{6 \text{ men}}$$

You take / I took the time it took 2 men to dig the whole which is 4 hours and I multiplied it by the number of men it took them.
Then the number I got I divided it with the 6 men.

Second test:

8. It takes 4 hours for 2 men to dig a hole. How long would it take 6 men to dig the hole of the same size?

A 1h30min

B $1\frac{1}{3}h$

C $\frac{3}{4}h$

D 1h33min

$$4 \text{ hrs} \div 2 \text{ men}$$

$$2 \text{ hrs / man}$$

In the initial test Abby's explanation and workings were up to standard, but she failed to interpret the time taken. She got the answer as 1.33333 hours which she interpreted as 1h33min - which was wrong. In the second test, she did not answer this question and what was done as her working was unsatisfactory. This indicated that time; ratio and proportion were one of her areas of weaknesses due to a lack of uniformity in her workings. Again, it emerges clearly that unstable knowledge is being exposed due to more recent learning about ratio.

Example J

Initial test:

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
- ☒ B 1 : 20 000
- C 1 : 200 000
- D 1 : 2 000 000

Second test:

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
- ☒ B 1 : 20 000
- C 1 : 200 000
- D 1 : 2 000 000

36 km : 1.8 cm Do not really know what I'm supposed to do here

In both tests Abby reached the same conclusion - 1: 20 000. In the second test, Abby admitted that she did not know what was expected of her. This explained why she did not provide any explanation in the initial test. Both her answers indicated that she had guessed, although she did not understand scale. Therefore Abby needed remedial lessons on scale.

4.6.2.2 Measures

This category consisted of two themes, namely convert and scale units correctly; and volume. The total mark was 9. Table 4.6.2.2 showed a template used in assessing individual learners' solution strategies for each theme in this category.

Table 4.6.2.2 Measures

Theme	Total marks
Learners can:	
-Convert units correctly	6
-Compute the volume of basic shapes, draw a diagram and label given parts, use the correct formula	3
Total	9

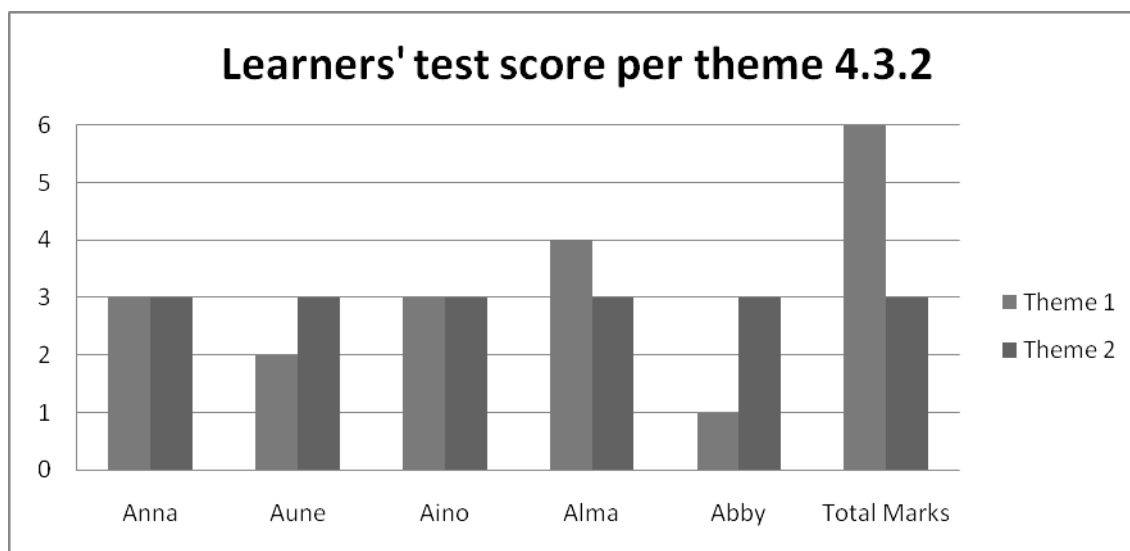
**Figure 4.6.2.2 Learners' theme scores for the Measures category**

Figure 4.6.2.2 above shows learners' test scores for each theme in this category. There were fewer questions covering themes in this category; therefore the results were not fully representative. More questions would be needed to make a fair judgement. Theme two had only one question and this was not sufficient to identify learners' areas of weakness.

In fact, all students obtained full marks for this question. *Aune* and *Abby* showed some weaknesses in the first theme (conversion and scaling of units) of this category and therefore they might require an introductory lesson on scaling and conversion of units. *Alma* did better than others in the first theme. While *Aino* and *Anna* scored more marks in the first theme, they also lack the basic skills of converting units and scaling, as demonstrated by their workings below.

Example K

Initial test:

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
B 1 : 20 000
C 1 : 200 000
D 1 : 2 000 000

d

0

Second test:

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
B 1 : 20 000
C 1 : 200 000
D 1 : 2 000 000

B-1 Multiply 36 km $\times 1000 = \frac{36000 \text{ km}}{1.8 \text{ cm}}$
 $= 20000$
Scale = 1 : 20 000 X

0

In both tests Anna's responses were the same - 1:20 000. As emerged from her working in the second test, Anna could not convert 36km into centimetres. The basic conversion of 1km = 100 000cm was lacking. Anna merely multiplied by a 1 000 as if she was converting to metres. The relation between length units was clearly a hindrance for Anna here.

Example L

Initial test

2. Anna suitcase has a mass of 0.275 kg. Convert the mass of Anna's suitcase into grams.

- A 275 000g
B 0.000 275g
C 27.5g
D 275g

$$\begin{aligned} 0.275 \text{ kg} \times 1000 &= 275 \text{ g} \\ 0.275 \text{ kg} \times 1000 &= 275 \text{ g} \end{aligned}$$

Second test:

2. Anna suitcase has a mass of 0.275 kg. Convert the mass of Anna's suitcase into grams.

- A 275 000g
B 0.000 275g
C 27.5g
D 275g

$$\begin{aligned} 0.275 \text{ kg} \times 10000 &= 275000 \text{ g} \\ 275000 \text{ g} & \quad \times \end{aligned}$$

In the initial test, Aune got the correct answer, but in the second test she used 10 000 as a conversion factor in her calculation, therefore she gave $0.275 \text{ g} \times 10000 = 275000 \text{ grams}$. Aune could also not multiply the above factors correctly. The conflicting conclusions Aune drew in the two tests provided evidence of her weaknesses in unit conversions. Therefore Aune too needed remedial classes on unit conversions.

Example M

Initial test:

2. Anna suitcase has a mass of 0.275 kg. Convert the mass of Anna's suitcase into grams.

- A 275 000g
B 0.000 275g
C 27.5g
D 275g

$$0.275 \times 1000 = 275 \text{ g}$$

Second test:

2. Anna suitcase has a mass of 0.275 kg. Convert the mass of Anna's suitcase into grams.

- A 275 000g
B 0.000 275g
C 27.5g
D 275g

X

First you already know that there's 100 g per kg therefore you have to multiply the kg item to grams ~~so that~~ in order to get the final solution

0

Aino did not know how many grams were in a kilogram, so instead of multiplying by a 1000 she used 100. In both the initial and the second tests, Aino's working was the same and she reached the same conclusion. She simply lacked basic unit conversion factors, thus it was judged that she needed remedial classes in this theme too.

Example N

Initial test

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
B 1 : 20 000
C 1 : 200 000
D 1 : 2 000 000

X

0

0

Second test:

20. On a map a distance of 36 km is represented by a line of 1.8 cm.
What is the scale of the map?

- A 1 : 2000
- ☒ B 1 : 20 000
- C 1 : 200 000
- D 1 : 2 000 000

36 km : 1.8 cm Do not really know what I'm supposed to do m
X

Abby failed to convert 36km into centimetres, which showed that the conversion of scale units was still a major problem among these learners. Although Abby did not understand the question about scale, she displayed some knowledge of the relationship between the ground and the map distance by writing 36 km: 1.8 cm.

4.6.2.3 Space and shape

There was only one theme in this category out of a total of 3 marks. Table 4.6.2.3 shows the template used to assess learners' solution strategies. Only two items were used in this theme.

Table 4.6.2.3 Space and shape

Theme	Total marks
Learners can:	
-work out the trigonometry ratio, display knowledge of angle properties	3
Total	3

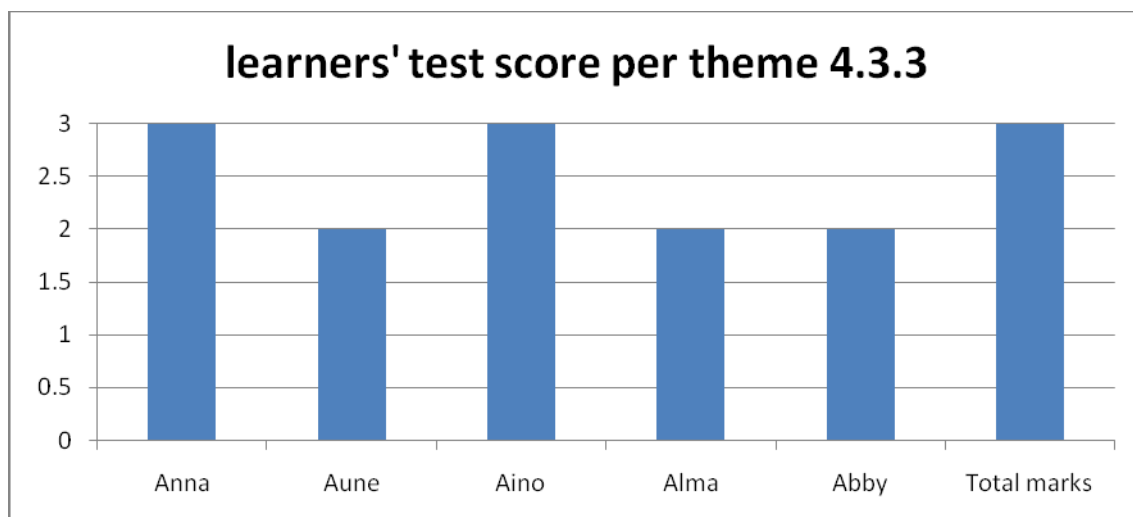


Figure 4.6.2.3 Learners' theme scores for the Space and shape category

The lowest mark obtained by an individual learner was two. The score for this theme was not viable for drawing any conclusion about learners' mathematical preparedness because the question coverage was too limited. Therefore I will not use any of this theme's items in the discussion.

4.6.2.4 Algebra

There were two themes which dealt mainly with algebra- related items. The total mark was 18. The template in table 4.6.2.4 was used to assess learners' solution strategies. Most research participants performed poorly in both themes in this category as indicated in figure 4.6.2.4.

Table 4.6.2.4 Algebra

Theme	Total marks
Learner can:	
-Evaluate and simplify; factorise and solve simple linear equation	14
-represent mathematical statements as an equation, transform formulae	4
Total	18

From the graph below, it was clear that all learners had some difficulties in algebra; hence all learners needed some remedial classes in either introductory or intermediate algebra. In introductory algebra learners are expected to evaluate algebraic expressions given the letter

values; to simplify and remove brackets, as well as to factorise algebraic expressions. In intermediate algebra, learners are expected to be able to solve simple linear equations, transform formulae and to represent mathematical statements as equations.

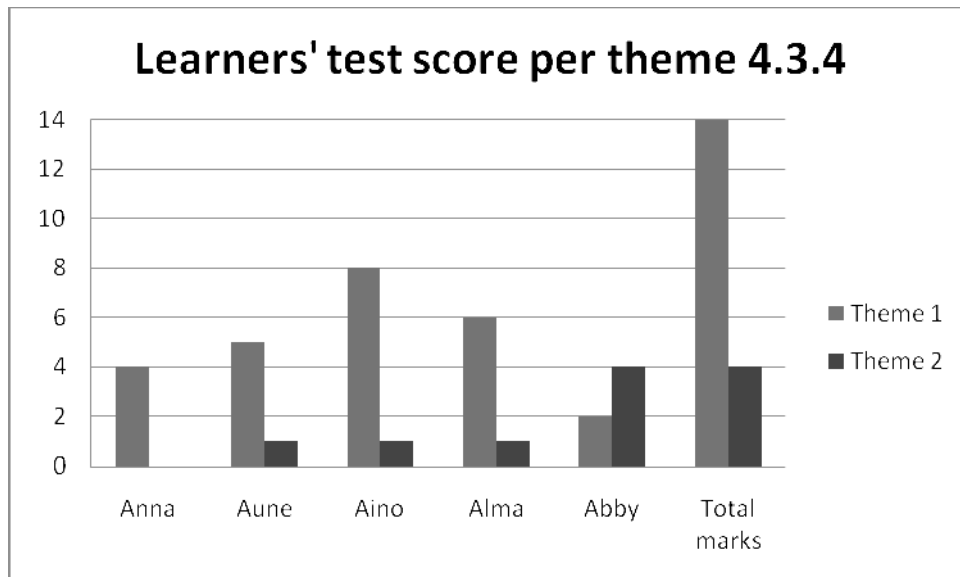


Figure 4.6.4.4 Learners' theme scores for the Algebra category

Anna got the second lowest mark of all in the first theme. This theme constituted more than two third of the total marks in this category. This indicated that Anna's basic algebra did not have a strong base. In the second theme, Anna did not also do well. This performance indicated that she needed introductory algebra classes. Below is an example of her working:

Example O

Initial test

11. Expand and simplify $3x(x-3)+2x(2x+1)=$

A $14x^3$

B $7x^2 - 7x$

C $7x^2 - 4x$

D $7x - 2$

$3x(x-3)+2x(2x+1)=$
 $3x^2 - 3x + 4x^2 + 2x =$
 $7x^2 - x$



Second test

11. Expand and simplify $3x(x-3) + 2x(2x+1) =$

- A $14x^3$
- B $7x^2 - 7x$
- ☒ C $7x^2 - 4x$
- D $7x - 2$

$$\begin{array}{l}
 3x(x-3) + 2x(2x+1) \\
 5x + 2x + 2x + -6 + 1 \\
 9x + -5
 \end{array}
 \quad
 \begin{array}{l}
 3x(x-3) + 2x(2x+1) \\
 3x(-6) + 4x + 2x \\
 3x + 4x - 6 + 2x \\
 7x^2 - 4
 \end{array}$$

In the initial test, Anna did not remove the brackets effectively. She failed to multiply each term inside the brackets with the common factor, and where she had tried an error occurred in the power of x . In the second test, the same error pattern occurred. She also could not multiply $3x$ by -3 because she got -6 as an answer. At the same time Anna did not add the indices when she multiplied $3x$ by x . It is interesting to note the improvement in Anna's working as she managed to do some multiplication in the second test whereas she could not do this in the first test. But the results show that her learning is not yet consistent. Therefore there is sufficient evidence from Anna's workings that she lacks basic algebraic skills, therefore she needs to attend introductory algebra lessons.

Aune also did not do well in both themes. The knowledge she displayed was not adequate for an extension of the Grade 11 curricula, therefore she needs introductory lessons in algebra. Look at her workings below.

Example P

Initial test:

13. Solve for x :

$$2(x-5) = -3x+5$$

- ☒ A -5
- B 3
- C -3
- D -15

$$\begin{array}{l}
 2(x-5) = -3x+5 \\
 2(-5-5) = -3x+5 \\
 -20 = -3x+5
 \end{array}$$

Second test:

13. Solve for x :

$$2(x - 5) = -3x + 5$$

- A -5
- ☒ B 3
- C -3
- D -15

$$\begin{aligned} 2(x - 5) &= -3x + 5 \\ 2x - 10 &= -3(3 + 5) \end{aligned} \quad \times \quad \bigcirc$$

In the initial test, Aune did not remove the brackets first before she attempted to collect like terms on one side. She also could not solve the resulting simple linear equation. In the second test, she does appear to have chosen the correct answer (3) even if the recording of the method is confused. Her substituting '3' on the right hand side implies that she solved this question using substitution (trying all answers to check which one works) and then tried to write out a solution that fitted with what she had found. But she didn't manage to do this, first battling to multiply the brackets on the left, and then confusing 'substitution' with 'simplification'.

Again here, we can see some development, both in terms of improved manipulation, but also in terms of knowing that checking by substitution is valid. Aune needs the basic knowledge of expanding and simplifying algebraic expressions in order for her to solve equations effectively. Hence, it emerges from Aune's solutions that her basic algebraic skills were poor and therefore she needs introductory classes on algebra.

Aino got the highest marks of all the participants in the first theme. Despite this, her marks were just over half of the total marks of this theme. Therefore Aino also needed intermediate lessons on algebra in order to grasp its finer details. In the second theme Aino, like the others, still exhibited a lack of algebraic skills, therefore she needed remedial lessons. Take a look at one example of Aino's workings below:

Example Q

Initial test

13. Solve for x:

$$2(x-5) = -3x+5$$

- A -5
- B 3
- ☒ C -3
- D -15

$$\begin{aligned} 2x-5 &= -3x+5 \\ 2x-3x &= -5+5 \\ 5x &= \end{aligned}$$

$$\begin{aligned} 2x-5 &= -3x+5 \\ -3 &= x \\ \therefore x &= -3 \end{aligned}$$

Second test

13. Solve for x:

$$2(x-5) = -3x+5$$

- A -5
- B 3
- ☒ C -3
- D -15

$$\begin{aligned} x+10 &= -3x+5 \\ -10+5 &= -3x \\ -5 &= -3x \\ \frac{-5}{-3} &= \frac{-3x}{-3} \\ x &= -15 \end{aligned}$$

multiply the ^{out} number ~~out~~ with those numbers in the bracket, after that correct the like terms and to be in the same side, as you are correcting that number must go along with its sign.

In the initial test Aino did not expand accordingly, she made $2(x-5)$ equivalent to $2x-5$. Because she could not expand successfully, her solution was no longer correct. In the second test Aino could expand correctly, but she could not solve for x successfully. From her explanation of the procedures used in solving the equation, it emerged that she had some misconceptions about how simple linear equations should be solved. For example, "... as you are correcting that number must go along with its sign". Therefore Aino would benefit from peer-tutoring or remedial classes in algebra.

Alma's marks were the second best among the participants in the first theme, but she did not do well in the second theme. Throughout her workings in both themes Alma displayed inadequate basic algebraic knowledge, and hence it was decided that she needed remedial classes in algebra. A view of some of her workings appears below:

Example R

Initial test:

13. Solve for x:

$$2(x-5) = -3x+5$$

- A -5
- ☒ B 3
- C -3
- D -15



3

$$3+3+5=3$$

Second test:

13. Solve for x:

$$2(x-5) = -3x+5$$

$$2(5-3+5) = -2+5$$

- A -5
- B 3
- C -3
- D -15

None



0

$$x=5$$

$$x = -3+5 - 2+5$$

Solving a simple linear equation systemically proved to be challenging for Alma. In the initial test Alma guessed the correct answer, because one can observe from her workings that they did not lead to the answer that she produced. In the second test Alma opted for substituting values to see if she could get to the answer instead of working systematically through the equation. And again she did not get the answer. Therefore Alma lacked basic skills for solving simple linear equations.

Abby's marks in the first theme were the lowest among the participants, but she scored full marks in the second theme. This showed that her knowledge about transforming formulae was adequate; hence she only needed remedial classes for the items similar to the ones in the first theme such as evaluation, simplification and factorisation of algebraic expressions as well as solving simple linear equations. Glance at one example of Abby's workings below:

Example S

Initial test:

13. Solve for x:

$$2(x-5) = -3x+5$$

- A -5
- B 3
- ☒ C -3
- D -15

9

$$\begin{aligned} 2(x-5) &= -3x+5 = 2x-5 = 5x \\ (2x-5) &= 2x = 2x+2x-5 = 4x-5 = 1 \end{aligned}$$

0

Second test:

(13) Solve for x:

$$2(x-5) = -3x+5$$

- A -5
- B 3
- C -3
- D -15

X

$$\begin{aligned} 7x &= 3+5 \\ 7x+3x &= 5 \end{aligned}$$

0

In the initial test Abby tried to solve the simple linear equation but she was unable to expand successfully. In the second test solving a simple linear equation remained a problem for her. Abby added 2 xs to 5 to get 7 xs; therefore she added unlike terms and she did not expand the brackets first. She also ignored the operation signs in her workings. All those facts provided sufficient evidence that Abby's basic algebra was not solid; therefore she needed introductory algebra classes.

4.6.2.5 Data handling and representations

This category consisted of only one theme with a total of 4 marks. Table 4.6.2.5 below represents the template used in the analysis of the test results.

Table 4.6.2.5 Data handling and representations

Theme	Total marks
Learners can:	
-Work out simple probability, in the simplest form -interpret the graph.	4
Total	4

Figure 4.6.2.5 shows a graphical representation of the learners' scores in the second test. Similar to 4.6.2.3 this category had insufficient items; therefore due to insufficient question coverage in the test there will be no discussion of solutions for this theme.

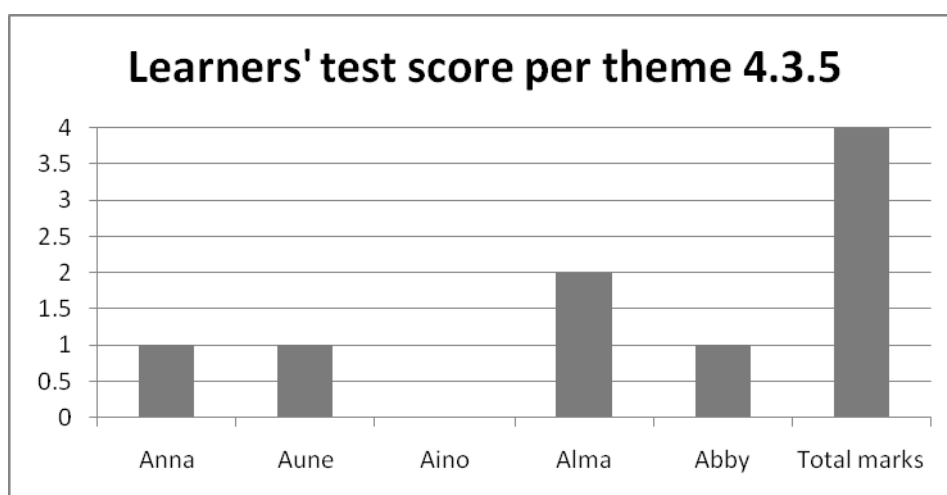


Figure 4.6.2.5 Learners' theme scores for the Data handling and representations category

4.7 Analysis of interview data

Learners were interviewed individually soon after completion of the second test. The interview data were discussed according to learners' areas of weakness in mathematics and with respect to the themes emerging from the discussion of learners' solution strategies.

4.7.1 Learners' mathematical competence

In their own words the learners highlighted the following:

Anna indicated that she did not understand factorisation even though she had covered it in Grade 10. The extracts below represent *Anna* sentiments about this area of weaknesses.

Extract 1

“(a pause) I am having trouble in factorization.
The algebra part is giving you problems?
Yes.”

‘A₁: 36-38’

Anna mixed up the terminology and vocabulary used in algebra. For example, she kept referring to factorisation as ‘solve’ which was only used in terms of equations (see transcript A₁: 44-46 for her explanation). It appears that Anna has conflicting understandings of these algebraic concepts. It also seems as if Anna could not differentiate between an algebraic expression and an equation, which is a common and well researched problem in learning algebra (Kieran, 1992).

Extract 2

“What exactly is the problem in algebra?

How to solve, I just don’t know (referring to question 10 in the second test: factorize completely $(4ax^2 - 10a^2x)$). The teacher said that for you to solve, first you look for the numerator which divides all numbers, like dividing into four and ten. From there; I don’t know the next step.”

‘A₁: 43-44’

It emerged from the interview extract 2 that Anna’s basic algebra is very weak because she uses wrong algebraic terminologies to explain other algebraic concepts. Her basic algebra is really confused as she was unable to distinguish between algebraic concepts.

Factorisation falls under the algebra category in my analysis, See Table 4.6.2.3 for themes in this category. Even though the teacher has explained how to factorize, Anna could still not grasp what she needed to do in order to obtain the number inside the brackets. It was clear that she had difficulties with the algebra-related items of the test. This interview data further justified the judgement based on her test solution strategies (see section 4.6.2.4 for her workings) – that Anna had misconceptions in algebra and needed introductory algebra classes.

Aune also identified algebra as her area of weakness in mathematics, especially concerning substitution and elimination. Below is an extract from the interview transcript.

“Do you feel you were well prepared upon entering grade eleven?”

I found out that I didn't have enough basic skills which are required before Grade eleven. Like something we are being taught now, were supposed to be taught in lower grades. Some topics were covered shallowly.

What are those things? Can you please be more specific?

For example algebra, substitution and elimination. Though they were taught in the lower grades, they were not taught thoroughly.”
‘A₂: 23-26’

Although substitution and elimination were not included in the test items, they would have fallen under the algebra category because they are used when solving simultaneous equations. Since Aune had difficulties in solving equations generally (refer to 4.6.2.4, example P), it was apparent that she needed remedial lessons in basic algebra.

Aino was not as forthcoming about her area of weakness in mathematics as Anna and Aune as seen in figure 4.6.2.3. This may be because Aino's basic algebraic skills were advanced when compared to the other participants. Therefore Aino could not pinpoint a specific area of weakness as she had some algebraic basics in place. Hence her interview did not yield anything relevant to this section.

Alma admitted that she had a problem with solving equations. See the interview extract below.

“That question was about.... Solving equations. I just don't understand. I just don't get the attention of solving this equation.

Didn't you do that in Grade ten?

I did it, but I just still have a problem with it.”

‘A₄: 22-24’

Once more, algebra proved to be a problem among the research participants. Though the topic was covered in Grade 10 Alma still had a problem with solving equations. In the test she left out that question without choosing an answer and her working was not satisfactory (see example R in part 4.6.2.4).

Abby identified angle properties, factorisation and expanding as her areas of weakness. Here follows some interview extracts for Abby: *Extract 1*

“Which part of Grade ten's mathematics do you think needed more attention?”

I think, the one that was neglected ... is the one where you can talk about angles, to find angles. And the other one ... mom ... ja! Factorising I didn't understand that very well in Grade ten mathematics.”

‘A₅:19-20’

Extract 2

“What do you mean by the term ‘expand’?”

I think like, you expand it, like you double it or something...

Mmh! Or you remove brackets?

You have to remove the brackets? So then, if I didn’t understand that, then I have to do that and the whole thing would be wrong. I didn’t do it at all. I was looking at it and I thought if I do something that is not it, I would think of myself as being dumb.”

‘A₅:27-30’

Like the other four learners, Abby also experienced difficulty with algebra-related test items. She could not understand a term like ‘expand’; therefore she left Question 11 undone. Abby also had a problem with angles, but in the second test she got the angles item correct.

4.7.2 Other themes emerging from learners’ interviews and solution strategies

Anna mixed up the mathematical terms most of the time, for example she used ‘mensuration’ for ‘solve numbers’; ‘solve’ for ‘factorisation’ and in her response to the test items she also mixed up ‘common factor’ for ‘lowest common multiple’(see example T in part 4.7.1). Look at the interview extract below:

What is mensuration? How do you understand the term?

Solve numbers.

Does it have to do with finding areas and volumes?

No, it is the one which you have to divide and you look for value of x and then you solve.”

‘A₁:23-26’

The extract above reveals that Anna did not fully understand the mathematical concepts and their meaning; therefore she kept mentioning one concept although though she meant another. Anna seemed to be familiar with these concepts, but she needed remedial classes on definitions or meanings of mathematical concepts in general so that she could become acquainted with the concepts and uses them correctly. Anna also seems to have a language problem which is a major stumbling block to her interpretation and understanding of these mathematical concepts.

Though *Aune* did not identify fractions as a problematic area for her, she could not explain her workings of the fraction problem in a satisfactory manner. See the interview extract below:

“Let’s check your working to question 6.

Evaluate: $\frac{1}{3} + \frac{1}{2}$ I have to add the numbers on top together and also get the sum of the number below. I don’t need to change the sign. The answer is $\frac{2}{5}$.”

‘A₂: 33-34’

The example below is the working to which she was referring when she offered the explanation above.

6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

A $\frac{2}{5}$

B $\frac{2}{6}$

C $\frac{5}{6}$

D $\frac{1}{6}$

$\frac{1}{3} + \frac{1}{2} = \frac{2}{6}$ X
you add

As a result of her explanation, I thought that Aune needed extensive introductory classes on fractions. Her conceptual understanding of fractions appeared way below the expected competence for her grade. Because for her to attain the competencies of Grade 11 mathematics; she needed to draw indirectly on her knowledge of fractions, its logic and its reasoning. In her case the required knowledge seems not to be there.

Abby did not understand the term ‘probability’ even though she had heard of the word before as the extract below revealed:

“There are 9 learners in a group, 6 are girls. What is the probability of picking a learner in a group who is a boy?”

Ok! I came across this question once, but guess I did something wrong of not asking what it really mean by probability. I think if ... I chose the ratio 6:3 because I worked it out, if you add 6 and 3 you get 9. That would be the sum of the ratio and then you get $\frac{6}{9}$ multiplied by the number of the learners then you get 6 which are already stated that they are girls.

Then I wanted to find out, I took $\frac{3}{9} \times 9$ pupils and then I got 3 boys. That made sense, If you add 6 to 3 that will give you 9 learners. But I was not so sure, ‘because there might be the right answer which is three, which also make sense, but I thought of taking 6:3 and also show my work on how I got the whole point.

If you have no clue of what is probability, how do you get to that answer?

I heard of it already but I never really asked what I am supposed to do when I am asked such a question.”

‘A₅: 42-43’

Although Abby had encountered probability earlier, she did not fully understand the concept of probability and what it entails. I commended her for acknowledging that she did not understand and for showing an eagerness to learn.

4. 8 Pulling the threads together

4.8.1 Strengths and grey areas of diagnostic testing

A number of benefits and pitfalls of diagnostic testing were identified in this research; therefore it is advisable when drawing conclusions, to take such shortcomings into consideration.

When used for the identification of learning difficulties, diagnostic testing does appear to provide some benefit for education delivery and the achievement of outcomes. It follows that diagnostic testing proves to be a useful tool for assessing learners' mathematical preparedness, so that the results may be used to improve the teaching and learning process specifically for remediation purposes. It also appears that an analysis of learners' solutions for each theme in each separate category provides greater insights into individual learners' mathematical preparedness. From this research, it is apparent that most of the participants have 'unstable knowledge' in most mathematical topics and concepts.

Some pitfalls in diagnostic testing also surfaced in this research. In particular, diagnostic test scores are not ideal for placement purposes because some participants (such as Anna) may be excluded unfairly. It also revealed that an analysis of learners' totals per category gives a vague idea of the categories where learners have 'unstable knowledge', but does not pinpoint specific areas where learners need assistance. Finally it emerged that in order to fully investigate learners' mathematical preparedness, there needs to be a good coverage of items in each category.

4.8.1.1 Readiness judgements: Competence verses test scores

Kilpatrick, et al. (2001, p. 10) emphasizes that mathematical competence should reflect what learners know and are able to do. According to Namibia. MBEC (1996, p. 9), mathematical competence is defined as "what a learner should be able to do as the outcome of teaching and learning", whereas PISA (2003), defined mathematical competence as "the fundamental mathematical knowledge and skills or an ability to mathematise" (p. 5).

It emerged from this study that test scores alone do not provide sufficient evidence of learners' mathematical competence. Mathematical competence does not only reside in high test scores. It was decided not to determine learners' mathematical competence by means of

the test scores only, as scores can be misleading, hence underestimating the true ability of the learners involved. For example, in this case Anna could have been excluded or regarded as *NR* due to test score boundaries set earlier, though in reality she fell into a *PR* group.

Therefore it was concluded from the evidence gathered in this research that diagnostic testing should not be used solely as a base on which acceptance or rejection decisions were to be made about any participants. Test scores, when coupled with analyses of learners' solution strategies provided sufficient evidence for a reasonable judgment of learners' mathematical preparedness. It appears that stimulated recall did not add much to this research, but the analysis of solution strategies yielded more.

Test attempts verses the analysis of solution strategies revealed that even the learners who were categorised as *R* or *PR* still had conflicting knowledge about specific mathematical content. This was supported by Habash, et al. (2006) who states that "there is a gap in knowledge and skills expected among learners in a secondary school, more pronounced in mathematics" (p. 101). This study bore testimony to the 'gap in mathematical knowledge' exhibited by these research participants. In conclusion, most research participants have yet to acquire stable knowledge regarding a number of topics in mathematics.

4.8.1.2 Remediation decisions based on the diagnostic test

Some remediation decisions were drawn from this diagnostic test analysis as follows:

Teaching

Theme totals analysis was used to identify topics in which learners needed remedial teaching. The topic identification enabled the teacher to prepare relevant items to cater for learners' individual needs. Furthermore it may also be used to determine the number of learners who need remedial classes. The use of diagnostic tests to assess the knowledge and understanding of learners has previously been described in literature (Armstrong & Croft, 1999; Batchelor, 2004). A range of uses of test results have been identified, including their use in the prediction of student knowledge from prior qualifications to assess their 'probable preparedness' and use in methods of delivery (Dixon, 2005).

The diagnostic test outcomes described in this study can be used both to influence the delivery of the curriculum and to assess the level of student understanding. As indicated by Bezuidenhout (1998), it is important that teachers "increase their awareness of students'

possible misconceptions and take them into account when developing suitable strategies for teaching”. Dixon (2005) also describes how diagnostic tests could be used “to predict areas that require particular attention and to develop appropriate teaching, learning and assessment strategies” (p. 55). Teachers’ judgements (refer to Table 4.3) are mainly based on how well they know their learners in the classes they have been teaching since the beginning of that year. Therefore teachers evaluate the learners’ general mathematics.

It emerges from both the solution strategies and some interview data that all participants experienced difficulties in the algebra related test items and also in number items, though the severity differs from learner to learner. Moreover, number sense is basically the backbone of algebra, so if a learner does not grasp number knowledge then it is expected that they will experience difficulties in algebra related problems.

The results indicated that Aune, Abby and Anna need remedial classes on a number of test themes because it surfaced from the data collected that they lack basic skills in introductory algebra and numbers in general. This is also supported in general by Tanner & Jones (2000, p. 147) and Batchelor (2004, pp. 69-74) who state that many children fail to grasp basic arithmetical concepts by junior secondary school. This study also reveals that this problem persists not only at junior secondary level, but up until the senior secondary school level where some learners still have poor foundations in mathematics. Namibia. Ministry of Education [MoE] (2007) confirms that most children leave school without the foundational skills and competencies they ought to have acquired. This study attests to that because some learners exhibit poor basic competence in mathematics by year eleven of their schooling.

It is judged that Aino and Alma need compensatory teaching to clear up a few misconceptions in mathematics to enable them to be mathematically-prepared to extend to the Grade 11 mathematics curricula. In agreement, Johnston-Wilder (1999) stated that “acquired misconceptions can have serious implications for pupils’ ability to comprehend new topics” (p. 105). Therefore it is a necessity to offer Aino and Alma compensatory classes to clear up their misconceptions.

Peer-to-Peer tutoring

It appears from the theme totals analysis that the diagnostic testing enabled teachers to identify learners whose mathematical basics are at a level which would enable them to be

used for peer-to peer tutoring. For example, in my research sample Alma and Aino did qualify for peer-to-peer tutoring, but before that they need compensatory classes on number sense and algebra like solving equations, and expanding in general.

Diagnostic testing may enable the researcher to group learners into different remedial classes, depending on the severity of their lack of mathematical preparedness at a particular grade level. Teachers and their students could use formative assessment tests to provide information which can be used in a remedial mode to modify the teaching and learning procedures in which they are engaged (Black & William, 1998, pp. 7-8).

4.8.1.3 How remediation decisions were made

(i) Analysis by category totals

In this analysis, learners' category totals are used. It emerges from the test items analysis done in 4.6.1, that the analysis of category totals did not give a good representation of learners' mathematical preparedness. A glance at figure 4.6.1 gives an impression that these learners were mathematically prepared for the Grade 11 curricula. In reality though, these learners were lacking some basic skills in mathematics, which were expected for their level.

Therefore it was not a good idea to analyse the diagnostic test results by category totals as it did not give a true reflection of the situation. Therefore, diagnosing the learners' areas of weakness in mathematics may not be effective when analysis by category totals is done. Analysis by category totals could have led to an incorrect judgement, because of its more general nature.

Learners could have been categorised as ready or partially ready even though they had some misunderstandings in basic mathematics. The analysis of learners' category totals did not identify specific areas of weakness for each individual learner. But it did give a general idea about which categories posed difficulties for learners, for example in algebra and data handling (see figure 4.6 under 4.6.1). This analysis did not identify which part of algebra was a problem. Therefore the analysis by category totals was not good for making remediation decisions.

(ii) Analysis of theme totals

When the analysis was done by theme totals in each category, more patterns emerged from the data. Theme by theme analysis was reasonable and it highlighted learners' specific areas of weakness in contrast to analysis by category totals, which portrayed only a vague idea of learners' preparedness in mathematics.

From the theme totals it appeared that most learners have difficulties in algebra-related and number-sense test items. As a result, I conclude that introductory classes in these areas should be offered to learners who appear to need help in areas such as factorisation, expanding, and solving equations, just to mention a few. Peer-tutoring can also be offered for those who are partially ready. Compensatory classes need to be offered to those learners who are ready but have some misconceptions and 'unstable knowledge' in mathematics.

It is concluded that it is worthwhile to use analysis of theme totals for such a diagnostic test, rather than analysis by category totals. The analysis of theme totals pinpoints that Anna, Aune, Aino and Alma have a very shaky foundation in fractions; therefore it is judged that these four participants need extra lessons on basic fractions. Goulding (1999) also supports the general conclusion that learners "have restricted or incomplete views of mathematical entities like fractions" (p. 40).

Furthermore Kilpatrick, et al. (2001, p.122) emphasize that when skills are learned without understanding, they were learned as tiny pieces of knowledge which are harder to connect to any new topics. Then learning new topics becomes difficult as there is no linkage between the previously learned concepts and skills and the new topic. These four learners' procedural fluencies needed to be improved.

Learners need to be able to justify and explain ideas in order to develop reasoning skills, and have a clear understanding of mathematical concepts and procedures (Kilpatrick, et al., 2001, p. 130). It emerged from this study that most of the research participants could not justify and explain certain mathematical concepts and procedures in their working (refer to 4.7), hence they display 'unstable' knowledge of mathematics.

4.8.2 Test questions coverage

Some categories in the diagnostic test had very few test items and because of this, learners' mathematical preparedness for these categories could not be interrogated accurately. For

example the space and shape category and the data handling and representation category contained far too few marks and items and no conclusion could be drawn about learners' mathematical preparedness in these categories. The space and shape category constituted 3 marks and data handling consisted of only 4 marks. The test also did not expose learners to a wide range of questions regarding the mathematical themes involved in these categories, and thus I judged that it was fair to exclude these two categories from the analysis of the learners' solution strategies.

For any diagnostic testing, a good representation of the range of questions in each category is necessary. A reasonable rule of thumb that emerges from this study is that each category needs to contain about 18 marks to include a good range of questions (at least 5 questions per category). It is only then that a good representation of a learner's solutions can be analysed and the category can be included in the analysis of the learner's mathematical preparedness. It also follows that more than one diagnostic test may be required for a full coverage of each topic.

4.8.3 Boundaries

Remediation decisions drawn under 4.8.1 B are based on the learners' marks per theme in each category and on the discussions of their solution strategies. The following boundaries are set regarding the conclusions drawn from the discussion of the solution strategies.

If a learner attained less than half of the total marks per theme, it was decided that such a learner needed remedial teaching on the specific theme. When learners attained at least half of the total marks per theme, compensatory teaching to correct a few misconceptions was recommended; whereas previously these learners were selected for peer to peer tutoring.

4.9 Conclusion

The following key insights arise from this research study:

- Diagnostic testing, when used in a constructive way can be effective for assessing learners' mathematical preparedness per phase. This study reveals that general conclusions can be drawn about the general mathematical preparedness of learners by

analysing learners' totals per category, but not their test scores. This is discussed in sections 4.5 and 4.6.1

The use of diagnostic tests to assess the knowledge and understanding of learners in specific subjects has previously been described in the literature (Armstrong & Croft, 1999; Batchelor, 2004). Dixon (2005) also identifies the use of diagnostic test results in the prediction of student knowledge from prior qualifications to assess their 'probable preparedness' and use in methods of delivery.

- It appears that diagnostic testing can be used to improve the teaching and learning process. Specific remediation decisions can be drawn, based on the analysis of the theme totals of each individual learner; and these decisions can benefit the education delivery and learning outcomes (see section 4.6.2 & 4.8.1B where different remediation decisions are drawn from the analysis of theme totals).

Furthermore, Black & William (1998) state that diagnostic testing can be used as an instrument to provide information which can be used in a remedial mode to modify teaching and learning procedures in which teachers and students are engaged.

- A good coverage of test items in each category is a necessity for an effective diagnostic test, because if this is not done one cannot fairly assess learners' mathematical preparedness in such categories. Sections 4.3.1, 4.3.2 and 4.3.4 provide good examples of categories where assessment of learners' mathematical preparedness can be done effectively. Therefore if a full coverage of each topic is required, more than one diagnostic test need to be conducted.
- Diagnostic test results should be used to identify learning difficulties, but not to place learners into particular categories which may disadvantage them. As happened in section 4.5, diagnostic tests should not be used for placement purposes as Anna could be excluded unfairly.

Diagnostic testing proved to be useful in assessing learners' mathematical preparedness by identifying learners' areas of weakness which could hinder their mathematics learning and performance. Taking into consideration the results of a diagnostic test could help teachers to cater for their learners who need remediation classes, as early as possible before extending the

mathematics curriculum. But setting and using diagnostic testing requires careful consideration because of the pitfalls highlighted by this research.

CHAPTER FIVE: CONCLUSION

5.1 Introduction

In this chapter I provide an overview of the study by reflecting on the research process, the research findings and the lessons learnt. Furthermore, I consider the limitations and potential value of this study and suggest areas for future investigation.

5.2 Critical overview of the study

In this study I set out to investigate Grade 11 learners' mathematical preparedness by assessing their mathematical competence. Through analysing learners' solution strategies in a diagnostic test it emerged that most participants have 'unstable knowledge' in mathematics and therefore they are partially ready to extend into the Grade 11 mathematics curriculum.

This research enabled me to work closely with five selected Grade 11 learners through the test and interviews. Hence I was able to interpret their solution strategies from the diagnostic test and draw inferences about their mathematical preparedness and thinking.

5.2.1 Overview of the research process

I used an interpretive case study which enabled me to obtain rich, deep data from this research. Though it was a small-scale case study and the results cannot be generalised, I was able to obtain valuable data from the learners which gives some insight into Grade 11 learners' mathematical preparedness in the selected school.

Being a novice researcher with no research experience, I experienced some difficulties in the process. For example, I waited for some time before I analysed the data collected, which made the process slow as I had to read through all the data to refresh my memory. I was also overwhelmed by the enormous volume of data at the start of the analysis. But in the end I managed to draw out themes, which provided a number of interesting insights.

The triangulation of the research data gathered by means of different research tools allowed me to assess and analyse the participants' mathematical preparedness by checking what they know and are able to do. At some stage and in some cases, the stimulated recall created depth and richness of the data and enabled me to get deeper insights into the learners' mathematical preparedness. Triangulation also helped me in validating the findings.

It is worth mentioning that during the research data collection process, I received good co-operation and full support from the school and the research participants.

5.2.2 Overview of the findings

The results and findings as presented in Chapter four reveal that though some Namibian learners had passed JSC examination, they still exhibited ‘unstable knowledge’ and lack of basic skills in mathematics.

Five major findings which may be of use to teachers to improve learners’ mathematical preparedness emerged from this study. Firstly, that diagnostic test results can be used to identify learning difficulties and possibly for placement purposes, but the sole use of a diagnostic test could lead to bad decisions. It arose from my research, that the test does not indicate the potential for learning and improvement; here Anna is a good example because of her great improvement between tests.

Secondly, the study revealed that the analysis of theme totals yields far richer and more meaningful data than the analysis of category totals. The analysis of category totals are too broad and so the variation of learners different strengths and weaknesses across the category averages out, resulting in intermediate scores which cannot be interpreted as ‘strong’ or ‘weak’ therefore disguising the learners’ areas of weaknesses. The findings from the analysis of theme totals can be utilised by teachers to assess the probable preparedness of their learners in terms of different important themes in each mathematical category.

Thirdly, it emerged from this study that remediation decisions can also be drawn from the thematic analyses of diagnostic test results. Teachers may use these findings to identify the exact mathematical topics or subtopics in which learners may need assistance. This could provide teachers with valuable information on the exact number of learners who need remedial classes, and also about learners who could be used for peer tutoring.

And fourthly, this study revealed that good coverage in the test is essential for an effective diagnostic test. That is, all categories in the test require a wide range of questions for effective analysis of learners’ mathematical preparedness.

Lastly, this study revealed that three of the five Grade 11 learners still possess ‘unstable knowledge’ in areas of mathematics that should have been mastered by this stage of their schooling.

5.2.3 Lessons learnt from this research

In conducting this research I have learnt many valuable lessons. From doing this study I have learnt to be patient and to manage my time more effectively. It enabled me to grow professionally and have self-discipline in completing any task. The interviews have enabled me to be attentive and a good listener. Though I approached this research as a novice researcher, I have gained experience and have greatly improved my interviewing skills. I am now better equipped to guide colleagues who are novice researchers.

The learners’ solution strategies provided me with deep and rich data, giving sufficient evidence with which to assess their mathematical preparedness. I have realised that learners’ mathematical preparedness needs to improve enormously for the effective extension of the Grade 11 mathematics curriculum.

5.4 Suggested areas for further investigations

With regard to the research findings from this study, I suggest that further investigations need to be carried out at the very beginning of the Namibian school year before any Grade 11 teaching commences. This is borne out by the finding that the research participants exhibited signs of ‘unstable knowledge’ in mathematics even after undergoing a trimester of Grade 11 teaching.

I would like to find out if my research finding regarding mathematical preparedness is indeed a general trend among Namibian Grade 11 learners. Therefore I suggest that further investigations should expand sample selection randomly in more Namibian secondary schools, before teaching commences for the year. This research also shows that diagnostic tests (with sufficient coverage) could be used as an effective tool for investigating this question at the level of important themes in different mathematical categories.

Further research could also be conducted to investigate the possible effects of implementing remediation decisions drawn from such diagnostic testing.

5.5 The potential value of the study

The research findings may be used by teachers to plan and improve their lesson delivery. These findings also inform teachers about learners' mathematical preparedness at year 11 of their education, painting a worrisome picture, given that all five participants passed the JSC mathematics examination. This study also reveals an understanding of the possible use of diagnostic testing, both its positive and negative effects in test results.

Although the scope and mode of this study is small, it has the potential to afford the Namibian Ministry of Education a glimpse into the Grade 11 learners' mathematical competence - which may enlighten the review of certain educational policies.

This study provides a contribution to the mathematical research field in Namibia which is overdue because very little research has been done regarding mathematics education in Namibia in the past.

5.6 Limitations of the study

The study is limited because it was a small-scale research project. Therefore, these findings cannot be generalised to all Grade 11 learners since only five learners were involved in this study and it was conducted within a limited time period. Due to the immense region size, the number of Grade 11 learners, and time constraints, the study had to be limited to only one secondary school in the Oshana Region.

The official procedure to obtain permission to conduct the research took a bit longer than expected; hence the research was conducted later than planned. Nevertheless, the data collected remains of great value to this study.

The inability to conduct the interviews after the first test, which led to the repetition of the test, was also one of the limitations of this study. Another limitation was the possible 'power' issue - due to the fact that I was a teacher and the head of department; and its effect of influencing learners to say what they thought I wanted them to say.

5.7 Conclusion

Through this study, diagnostic testing proves to be useful in assessing learners' mathematical preparedness by identifying learners' areas of weakness which hindered their mathematics learning and performance. Taking into consideration the results of a diagnostic test could help teachers to cater for their learners, who need remediation classes as early as possible before extending the mathematics curriculum. But setting and using diagnostic testing requires careful consideration because of possible drawbacks highlighted by this research.

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APPENDICES

APPENDIX A: DIAGNOSTIC TEST (FINAL)

MATHEMATICS DIAGNOSTIC TEST

MAY 2008

LEARNER'S NAME:

Grade 11.....

Region.....

Time: 2h



INSTRUCTIONS TO PARTICIPANTS

This question paper consists of 23 questions.

Answer all questions.

There are four possible answers labelled A, B, C and D per each question.

Use a pencil to circle the letter of your choice.

Show all your working in the box below each question.

1. Written in a standard form 0.00816 is

- A 816×10^{-5}
- B 8.16×10^3
- C 8.16×10^{-3}
- D 0.816×10^2

2. Anna suitcase has a mass of 0.275 kg. Convert the mass of Anna's suitcase into **grams**.

- A 275 000g
- B 0.000 275g
- C 27.5g
- D 275g

3. Which one of the following numbers is the smallest

60% 0.67, $\sqrt{0.38}$ $\frac{2}{3}$

A $\frac{2}{3}$

B 60%

C $\sqrt{0.38}$

D 0.67

4. 2.8971 m written correct to two decimal places and two significant figures is:

2 decimal places.

2 significant figures

A 3.10m

2.90m

B 2.9m

2.89m

C 2.897m

2.90m

D 2.90m

2.9m

5. Make x the subject of the formula

$$\frac{x}{a} + b = c$$

A $x = ac + bc$

B $x = \frac{c-b}{a}$

C $x = ac - b$

D $x = a(b - c)$

6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

A $\frac{2}{5}$

B $\frac{2}{6}$

C $\frac{5}{6}$

D $\frac{1}{6}$

7. Ms Nakamwandi left N\$ 14 000 to be divided among her sons; Sam and John, upon her death. Sam should get the largest amount when the money is divided in the ratio 4:3.

How much would John get?

- A N\$ 9 000
- B N\$ 8 000
- C N\$ 6 000
- D N\$ 10 500

8. It takes 4 hours for 2 men to dig a hole. How long would it take 6 men to dig the hole of the same size?

- A 1h30min
- B $1\frac{1}{3}h$
- C $\frac{3}{4}h$
- D 1h33min

9. Evaluate $3xy - 2x^2$ when $x = -1$ and $y = 2$.

- A -4
- B -6
- C -8
- D 10

10. Factorize completely $4ax^2 - 10a^2x$

- A $2a(2ax^2 - 5a^2x)$
- B $2ax(2x - 5a)$
- C $ax(4x - 10a)$
- D $4ax^2(1 - 2x)$

11. Expand and simplify $3x(x - 3) + 2x(2x + 1) =$

- A $14x^3$
- B $7x^2 - 7x$
- C $7x^2 - 4x$
- D $7x - 2$

12 Simplify

$$\frac{2x^2}{10y^2} \div \frac{5x}{5y}$$

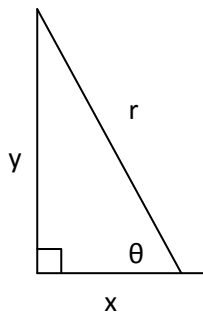
- A $\frac{2x^3}{5}$
- B $\frac{2x^3}{20y^3}$
- C $\frac{10x^3}{50y^3}$
- D $\frac{x}{5y}$

13. Solve for x :

$$2(x - 5) = -3x + 5$$

- A -5
- B 3
- C -3
- D -15

14. Find the ratio for $\tan \theta$.



- A $\frac{x}{r}$
- B $\frac{y}{r}$
- C $\frac{x}{y}$
- D $\frac{y}{x}$

15. The cash price of a TV set was N\$6999. If a customer is buying on hire purchase, a deposit of N\$1500 should be paid and N\$499 monthly instalments for 12 months.

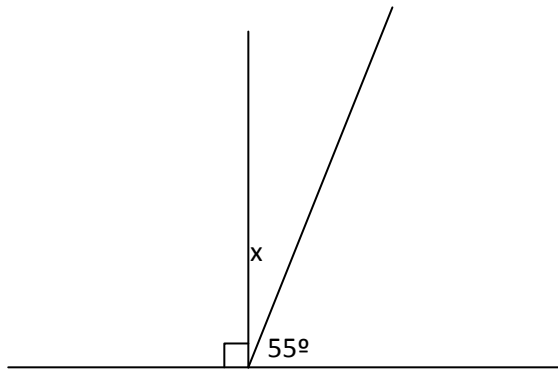
How much **more** does the customer buying on hire purchase pay?

- A** N\$ 8 488
- B** N\$ 5 988
- C** N\$ 15 487
- D** N\$ 489

16. A half-closed cylinder is 7cm high and has a radius of 3cm. Calculate its volume.

- A** 19.792cm^2
- B** 131.95cm^3
- C** 197.92cm^3
- D** 1.979cm^3

17. Find the value of x .



A 45°

B 25°

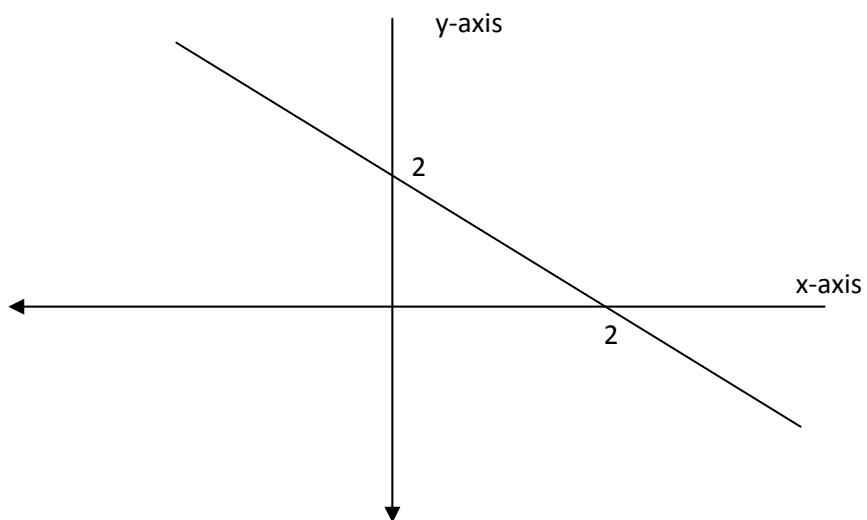
C 35°

D 55°

18. There are 9 learners in a group, 6 are girls. What is the probability of picking a learner in a group who is a boy?

- A** $\frac{6}{9}$
- B** $\frac{1}{3}$
- C** 3
- D** 6:3

19.



Work out the gradient of the given line in the graph above.

- A** 2
- B** -1
- C** -2
- D** 1

20. On a map a distance of 36 km is represented by a line of 1.8 cm.

What is the scale of the map?

- A** 1 : 2000
- B** 1 : 20 000
- C** 1 : 200 000
- D** 1 : 2 000 000

21. How many prime numbers are there between 30 and 40?

- A** 0
- B** 1
- C** 2
- D** 3

22. Find the lower bound and upper bound of 10.4m.

A $10.45m \leq H < 10.5m$

B $9.45m \leq H < 10.5m$

C $9.35m \leq H < 9.45m$

D $10.45m \leq H < 10.35m$

23. I start with a number x , then square it, multiply by 3 and finally subtract 4.

the final result is:

A $(3x)^2 - 4$

B $(3x - 4)^2$

C $3x^2 - 4$

D $3(x - 4)^2$

Thank you for taking part in this exercise!

APPENDIX B: DIAGNOSTIC TEST (FINAL)

MATHEMATICS DIAGNOSTIC TEST

MAY 2008

LEARNER.'S NAME:.....

Grade 11.....

Region.....

Time: 2h



INSTRUCTIONS TO PARTICIPANTS

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C 2.897m

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D 2.90m

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6. Evaluate: $\frac{1}{3} + \frac{1}{2}$.

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B $\frac{2}{6}$

C $\frac{5}{6}$

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7. Ms Nakamwandi left N\$ 14 000 to be divided among her sons; Sam and John, upon her death. Sam should get the largest amount when the money is divided in the ratio 4:3.

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- A -4
- B -6
- C -8
- D 10

10. Factorize completely $4ax^2 - 10a^2x$

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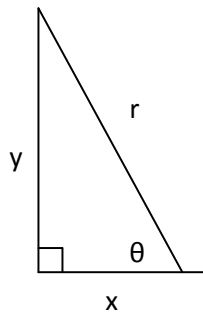
- A $\frac{2x^3}{5}$
- B $\frac{2x^3}{20y^3}$
- C $\frac{10x^3}{50y^3}$
- D $\frac{x}{5y}$

13. Solve for x :

$$2(x - 5) = -3x + 5$$

- A -5
- B 3
- C -3
- D -15

14. Find the ratio for $\tan \theta$.



A $\frac{x}{r}$

B $\frac{y}{r}$

C $\frac{x}{y}$

D $\frac{y}{x}$

15. The cash price of a TV set was N\$6999. If a customer is buying on hire purchase, a deposit of N\$1500 should be paid and N\$499 monthly instalments for 12 months.

How much **more** does the customer buying on hire purchase pay?

A N\$ 8 488

B N\$ 5 988

C N\$ 15 487

D N\$ 489

16. A half-closed cylinder is 7cm high and has a radius of 3cm. Calculate its volume.

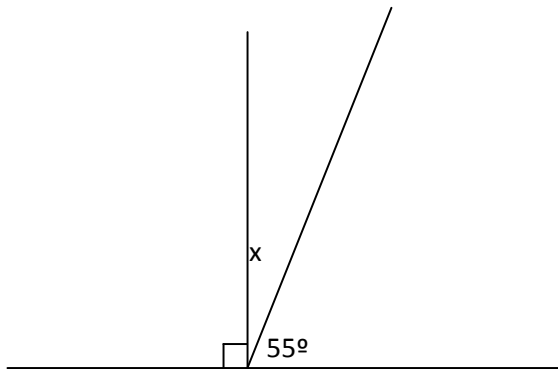
A 19.792cm^2

B 131.95cm^3

C 197.92cm^3

D 1.979cm^3

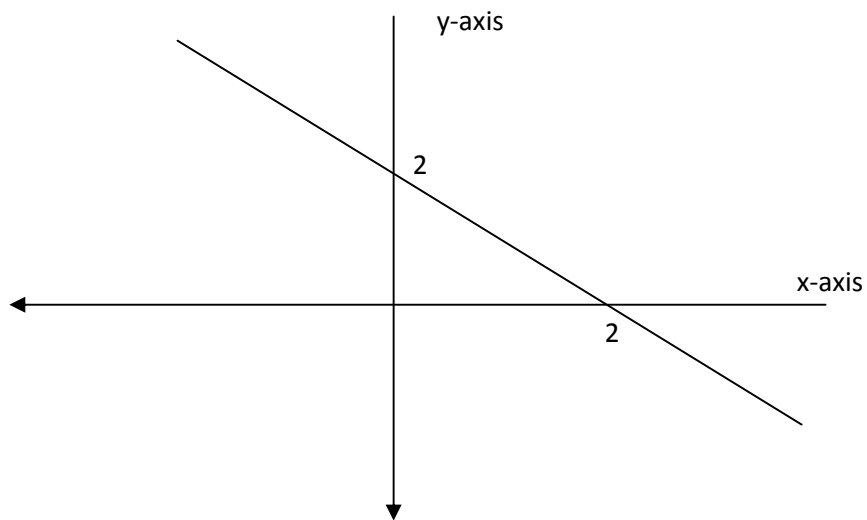
17. Find the value of x .



18. There are 9 learners in a group, 6 are girls. What is the probability of picking a learner in a group who is a boy?

- A $\frac{6}{9}$
- B $\frac{1}{3}$
- C 3
- D 6:3

19.



Work out the gradient of the given line in the graph above.

- A** 2
- B** -1
- C** -2
- D** 1

20. On a map a distance of 36 km is represented by a line of 1.8 cm.

What is the scale of the map?

- A** 1 : 2000
- B** 1 : 20 000
- C** 1 : 200 000

D 1 : 2 000 000

21. How many prime numbers are there between 30 and 40?

A 0

B 1

C 2

D 3

22. Find the lower bound and upper bound of 10.4m.

A $10.45m \leq H < 10.5m$

B $9.45m \leq H < 10.5m$

C $9.35m \leq H < 9.45m$

D $10.35m \leq H < 10.45m$

23. I start with a number x then square it, multiply by 3 and finally subtract 4.

the final result is:

A $(3x)^2 - 4$

B $(3x - 4)^2$

C $3x^2 - 4$

D $3(x - 4)^2$

Thank you for taking part in this exercise!

APPENDIX C: INTERVIEW SCHEDULE

DATE TIME

Topic: Investigation of Grade 11 learners' mathematical preparedness at a selected
Namibian school. A case study.

All answers will be utilized anonymously in the research report. However, provide me with
some personal data that could help me with analysis.

Briefly tell me about your schooling experience before coming to this school.

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.....
.....
.....
.....

Tell me about your experience in mathematics as a school subject.

.....
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.....
.....

1. How did you find the test?

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.....
.....
.....

2. Which parts of the test were less challenging to you?

.....
.....
.....
.....

3. Was there any part of the test more challenging than others?

.....
.....
.....
.....

4. What was your reaction to the test upon receiving the question paper/ How did you feel, when you were taking the test?

.....
.....
.....
.....

5. Can you explain to me your working/ answer to question?

.....
.....
.....
.....

6. Have you identified the topics or themes from where the questions had been based on? Give examples.

.....
.....
.....

Thank you for your time!

APPENDIX D: INTERVIEW TRANSCRIPTS

Transcript for Interview 1: Anna

04/09/2008

1	I	Briefly tell me who you are?
2	L ₁	I am Anna... (speaking softly)
3	I	Would you please speak up!
4	L ₁	I am Anna.
5	I	Is that all? Apart from your name, please tell me about yourself.
6	L ₁	I am Anna. I am schooling at Mooti Secondary School. I am from Ongenga in Endola Circuit, Ohangwena Region. On the part of Mathematics like you are saying, I like mathematics as it taught me a lot of things about calculations.
7	I	What are exactly are those things?
8	L ₁	Like money matters.
9	I	Would you please be specific?
10	L ₁	Like, you will know how much money you have and if you are selling, you will be able to give the correct change.
11	I	So, mathematics is used in your daily life?
12	L ₁	Mmh!
13	I	Would you tell me about your schooling experience, starting from kindergarten up to date?
14	L ₁	I started kindergarten at Omundundu up to Grade 1. Where Grade 2; I went to Ongenga, where I attended from Grade 2 from 1999 to 2007.
15	I	Mmh how was mathematics then?
16	L ₁	At first, I felt mathematics was difficult, and then I was told that I should not be afraid if I am in mathematics classroom. I should not be afraid when the teacher is teaching, but I should listen carefully, and take it easy.
17	I	Ok! Any inspiration from something or somebody, which inspire you to view mathematics the way you do now?
18	L ₁	My mathematics teacher for Grade 8-9. Sometimes, I didn't know how to operate a scientific calculator, but I know only the number buttons and not other buttons

		<p>above the numbers and why they are there. That teacher told me the purpose of those buttons and how to use them; she also taught me how to correct an error on a calculator.</p> <p>She also taught me that when I am calculating, I should write down a formula first before, you explain or show how you get to the answer. Do not just write down an answer because workings need to be shown on order to earn marks. For example, if a question has 3 marks, if you did not show your working, then you will only get 1 mark.</p>
19	I	So it is important to show or explain your working. Ok! How is Grade 11 mathematics?
20	L ₁	It is somehow..., sometimes it is easy, but others challenging especially the new topics which are introduced are sometimes difficult.
21	I	New topics? Can you give an example?
22	L ₁	Like ... finding 'theta', mensuration.
23	I	What is mensuration? How do you understand the term?
24	L ₁	Solve numbers.
25	I	Does it have to do with finding areas and volumes?
26	L ₁	No, it is the one which you have to divide and you look for value of x and then you solve.
27	I	<p>Ok!</p> <p>Do you feel you were mathematically well-prepared from Grade 10?</p>
28	L ₁	I feel I was well-prepared.
29	I	Was there any topic, you were not comfortable with?
30	L ₁	No.
31	I	How was the test?
32	L ₁	It was okay, I enjoyed it, but ... But there were some tricky questions.
33	I	Like which one?
34	L ₁	Can I page through?
35	I	Yes, you can.

36	L ₁	(a pause) I am having trouble in factorization.
37	I	The algebra part is giving you problems?
38	L ₁	Yes.
39	I	Did you cover algebra in Grade 10?
40	L ₁	Yes, we did, but not thoroughly.
41	I	Do you ask you teacher when you did not understand?
42	L ₁	Yes.
43	I	What exactly is the problem in algebra?
44	L ₁	How to solve, I just don't know (referring to question 10 in second test : factorize completely $4ax^2 - 10a^2x$). The teacher said that for you to solve, first you look for the numerator which divides all numbers, like dividing into for and ten. From there; I don't know the next step.
45	I	So, you don't know what you should do to get what is inside the brackets. Does the teacher tell you how to get the answer?
46	L ₁	Sometimes, when he is writing on the board, you just find ..., first he ask the class, the number which divides into ten, then from there I am lost. I don't know how he gets to the answer.
47	I	Would you please explain your workings to question 22? How did you conclude that your answer is B?
48	L ₁	I get it as when we were taught this year, our teacher said, to find a lower bound, you have to divide first the given number to the nearest number and divide it by two. You get an answer. The answer you get, minus it from a given number to get a lower bound. Add the answer to a given number to get an upper bound. Like here (pointing to the working), 10.4 I added 5... because 5 is here.
49	I	Above 10 or 4?
50	L ₁	Above 4.
51	I	Ok. If you were asked to advice mathematics teacher, what would you say?
52	L ₁	First 10 minutes of the lesson, the teacher should pose questions to learners while their books are closed.
53	I	The questions to be asked about that day 's lesson or the previous lesson

54	L ₁	It is the previous lesson. What he will teach, he just give one example and give you a chance to the learners to solve exercise on the board. Because some of the learners could not understand, but they are saved when other learners do ask questions. Some learner can leave the class without saying anything.
55	I	If others are asking questions, does it affect other learners positively?
56	L ₁	Yes, and also to write on the board. Because every time one is going to write on the board, one is afraid that if I make a mistake, I will be send back again to redo.
57	I	Thank you for your time'

Transcript for Interview 2: Aune

04/09/2008

1	I	Briefly tell me about yourself.
2	L ₂	My name is Aune, my surname is Moongo. I am 17 years old. I am from Oshiko and currently... currently I am doing Grade 11 at Mooti Secondary School. I like to play netball and to read novels. When I am finished with Grade 12, I want to... I want to further my studies at UNAM, so that I would become a doctor. I just found out that there is a lack of doctors in our country.
3	I	It is true, we need doctors. How was mathematics throughout your schooling?
4	L ₂	From all those grades, mathematics was ok. Up to now I am coping though there are a few things which are giving me trouble. As you go up the grades, so is the level of difficulties in mathematics.
5	I	What exactly that is getting tough? Can you be specific?
6	L ₂	There is ... is mensuration which I don't really understand.
7	I	What is mensuration?
8	L ₂	It has to do with measuring cylinders and volumes.
9	I	Do you seek assistance from your teachers?
10	L ₂	Yes.
11	I	Did the teacher respond to your questions?
12	L ₂	Yes.
13	I	Which symbol did you get in Grade 7 mathematics?
14	L ₂	I got C-symbol.
15	I	And in Grade 10?
16	L ₂	I got a D-symbol.
17	I	Who or what inspire you with respect to your current view of mathematics?
18	L ₂	Yes, when I was in Grade 10, there was one student teacher from UNAM, who helped us out every weekend. She gave us quizzes of multiples of 1 to 10, which we have to complete without using a calculator. The first person to finish will be given a gift.

19	I	Do you believe that if learners are awarded prizes, they will do well in mathematics?
20	L ₂	Yes.
21	I	Ok. How is grade 11 mathematics?
22	L ₂	Grade 11 mathematics is okay; up to here I did not experience any problems.
23	I	Do you feel you were well prepared upon entering grade 11?
24	L ₂	I found out that I didn't have enough basic skills which are required before grade 11. Like something we are being taught now, were supposed to be taught in lower grades. Some topics were covered shallowly.
25	I	What are those things? Can you please be more specific?
26	L ₂	For example algebra, substitution and elimination. Though they were taught in the lower grades, they were not taught thoroughly.
27	I	How was the test?
28	L ₂	The test was okay, except for some few questions which were a bit challenging.
29	I	Which question exactly?
30	L ₂	Like this question (referring to question 22) of the bound. We were taught that the number you get to the nearest, you divide it by 2. But here we are not given to the nearest but 10.4.
31	I	So it does not correlate to what you have learnt? Apart from the lower bound, what else was bothering you?
32	L ₂	(paging through) Everything was ok?
33	I	Let's check your working to question 6'
34	L ₂	Evaluate: $\frac{1}{3} + \frac{1}{2}$. I have to add the numbers on top together and also get the sum of the number below. I don't need to change the sign. The answer is $\frac{2}{5}$.
35	I	If you were a mathematics teacher, what would you do differently?

36	L ₂	The teacher should give mathematic sets for each learner. When a chapter is finished, give a topic test, if learners perform well that is a proof that the teacher accomplish the lesson's goal. If learners fail, then the teacher has to revise the lesson and retest again.
37	I	Thank you for your time.

Transcript for Interview 3: Aino

04/09/2008

1	I	Briefly tell me about yourself.
2	L ₃	About myself? My name is Aino Niingo. My previous school was Oingo Combined School, where I schooled from grade 1 to grade 10. And since I was grade 10, my favourite subject was mathematics because our teacher used to tell us that if you do not have mathematics nowadays you won't be admitted at any institution.
3	I	Mmh, that influence you to like mathematics.
4	L ₃	Yes.
5	I	Ok! Which symbol did you get in Grade 7 mathematics?
6	L ₃	Apparently I got a B,... I think it is a B.
7	I	And in grade 10, which symbol for mathematics?
8	L ₃	I got an A.
9	I	How are coping with grade 11 mathematics?
10	L ₃	Grade 11 mathematics is really interesting. Somehow easy, somehow difficult. But at least I am enjoying it.
11	I	Which part you feel is more difficult in grade 11?
12	L ₃	The part that are difficult to me ... is like elimination, but not really difficult and subst..., no substitution, only elimination.
13	I	One way of solving simultaneous equations?
14	L ₃	Mmh!
15	I	Ok, do you feel you were well prepared in mathematics upon entering grade 11?
16	L ₃	Yes, everything was comfortable.
17	I	Every part you think you were ready?
18	L ₃	Yes.
19	I	Generally, how was the test?

20	L ₃	Ja, it was good.
21	I	Anything which was more challenging?
22	L ₃	Everything was just fine.
23	I	Would you please explain your working to question 5? First read the question aloud.
24	L ₃	Make x the subject of the formula $\frac{x}{a} + b = c$.
25	I	What does the statement means?
26	L ₃	Like this letter x would be the subject, it would be the first letter of the equation.
27	I	Mmh, leaving x alone on one side.
28	L ₃	Mm!
29	I	Can you explain your working then and how reach to the answer?
30	L ₃	I think this is the answer cause, if you want to get rid of x , for it to be a subject... I think you have to multiply on both sides with x . And then she will come on the other side of this letter and then x will become the subject of the formula. And both equation will be divided by a .
31	I	Divide by a, ok! If you were a mathematics teacher, what advice would you give to your students?
32	L ₃	If I was a mathematics teacher, for instance, I am teaching mathematics in a class, I wi... after each lesson I will give the learners exercises to test how they are performing. After each chapter I have to give them a topic test so that I will check how far they understand, their standing or position. If they fail to do so, then I have to repeat the chapter.
33	I	Thank you for your time!

Transcription for Interview 4: Alma

04/09/2008

1	I	Briefly tell me about yourself.
2	L ₄	I am Alma and Iita is my surname. I am doing grade 11 here at Mooti Secondary School. All the way from Oshimba Combined School, Endola Circuit, where I school since grade 1 to 10.
3	I	Tell me about your mathematical experience through schooling.
4	L ₄	Since grade 5, mathematics was too difficult for me. But by the time I got in grade 8, my mathematics teacher, let me say I liked her and she taught well. And she made me aaa.... To like mathematics more than before. I keep on doing and liking mathematics and put on (inaudible). Till now I like mathematics.
5	I	Mmh. Can you share with me what was really difficult at those early grades?
6	L ₄	I don't think it was so difficult, but the thing is just that I hate the subject by that time. I just hated the subject for no reasons
7	I	There was nothing wrong or difficult about the subject, was it just your attitude towards mathematics then?
8	L ₄	Mmh! (nodding)
9	I	Oh! Which symbol did you get in grade 7 mathematics?
10	L ₄	I can remember, by the third term I got a D symbol.
11	I	A D-symbol. And in grade 10?
12	L ₄	In grade 10, it is a B.
13	I	Oh, you have improved! How are you coping with grade 11 mathematics?
14	L ₄	So, up to now, it is somehow good. Because I don't think I have so many problems with it.
15	I	Do you feel you were mathematically well-prepared upon leaving grade 10?
16	L ₄	No something, I was so much prepared.
17	I	How was the test?

18	L ₄	Somehow good.
19	I	Anything challenging? (learner page through)
20	L ₄	Ja!
21	I	Like?
22	L ₄	That question was about.... Solving equations. I just don't understand. I just don't get the attention of solving this equation.
23	I	Didn't you do that in grade 10?
24	L ₄	I did it, but I just still have a problem with it.
25	I	So you left the question like that?
26	L ₄	Ja.
27	I	Apart from question 13, anything else?
28	L ₄	Nothing else.
29	I	Can you explain your workings to question 18?
30	L ₄	I get like, 9 learners in a group, 6 are girls and 3 are boys. The question is: what is the probability of picking a learner who is a boy? 9 learners I minus 6 girls and then remains 2 boys... (hesitatingly she continued) 9 minus 6 remains 4,... 4 boys. 3 boys I mean. Those 3 boys I make it out of 1.
31	I	Out of....?
32	L ₄	Out of 1.
33	I	Is it 3 over 1 or 1 over 3?
34	L ₄	3 over 1.
35	I	Hmmm, and the answer will be?
36	L ₄	(laughing) I think it is 1 over 3.
37	I	Why did you change your mind?
38	L ₄	Cause (kept quiet for sometime) 6 girls minus
39	I	Where does this 1 come from?
40	L ₄	It is like ... I minus the number of boys, in the number of girls I minus the number

		of boys. Then I get 2.
41	I	Let's look at your workings to question 22.
42	L ₄	Mmm, (a long sigh). It is like a decimal. I used $\frac{1}{10}$ to make 0.01. Then from 10.45, I minus 0.1 to get the lower bound, I mean from 10.4. To get the upper bound I add 0.1 to 10.4.
43	I	What advice would you give to the mathematics teacher?
44	L ₄	I will advice him/her, in order to cope for his subject to be passed so well performed, he should give more exercises and tests, to make sure the learners understand his/her subject.
45	I	Thank you for your time!

Transcription of Interview 5: Abby

04/09/2008

1	I	Briefly tell me about yourself.
2	L ₅	I am Abby Davids. I am 17 years and in grade 11. And I ... I am kind of ... into study, kind of get tough especially in mathematics and some stuff. But I realize that mathematics is not really tough as we use to think. It needs concentration and to practice more, then it would become better. We become lazy sometimes to study but we know it is for our own good, so we do it.
3	I	Ok! Mm, tell me about your schooling experience, starting with kindergarten up to now.
4	L ₅	<p>Ja! I went to kindergarten several times because I couldn't stay at home alone.</p> <p>So I went with my sisters whenever they went I also went, until I reached that point of really attending kindergarten. And then ... I went in grade 1 at Oshako, then I have to repeat as I got sick. When we moved from Ongwediva to Oshakati I repeated grade 1 again and from there to grade 3.</p> <p>I was a good student and they wanted me to ... to go already to grade 5, but then I was too young. My father was happy about it, but when he came to find out that I was too young for the grade, he got so sad. Anyway I made it through to grade 4 and things were fine. I loved science subjects very much and in grade 5 I was doing well until grade 7 to 8. And then in grade 10, life went through... much focusing on different stuff, but I managed to pass grade 10 with 35 points. And here I am in grade 11, trying to make things better.</p>
5	I	Ok! In grade 7, which symbol did you obtain in mathematics?
6	L ₅	Ja ...! I got an A-symbol, because I was in the same class with this other boy, he came at the first position in the first term and I came second. I came second the first term, but in the second term and third term I manage to have the first place in the class.
7	I	Wow! What symbol did you get in mathematics in grade 10?
8	L ₅	In mathematics, I got a B.
9	I	Got a B?
10	L ₅	Ja, we were fighting but then I got manage to get a B.

11	I	Ok! What really do you enjoy in mathematics?
12	L ₅	In mathematics, I enjoy the ability to kind of ... to think about it and then you get that answer. And I also enjoy how sometimes an equation can be challenging to you, you just face it and you are there, like am I really suppose to do this? I mean the x plus y , where I am going to use them and stuff. And I also enjoy that time when you really to think of yourself kind of do it and until the last point you get the correct answer. You feel proud of yourself that I can actually use my mind and get something useful.
13	I	Ok! Anything or anybody that influence you on how you view mathematics today?
14	L ₅	Mm, my brother a lot, he is in UNAM, he is doing his last year. He is one person who says ... he told me I remember on my 16 th birthday he gave me a book, and then we were talking as I trust him a lot and we have a good connection. He told me Abby, if you tell yourself that mathematics is hard, because I used to tell him, brother, mathematics... I cannot do it in mathematics, it is a little bit tough and I cannot get what I target. He told me, if you keep telling yourself, mathematics is tough, it gonna get tough and it will keep on being tough. But if you tell yourself I can do it, then you make it simple, step-by-step, you will get it and you will be as better as your mathematics teacher and as better as anyone who do great in mathematics. And from there on, I learn that it is better to have positive thought about it and I will also have positive marks.
15	I	Oh! What an inspiration! How are you coping with grade 11 mathematics?
16	L ₅	Mm ... grade 11, when I first came in I thought, it was ... ok, back then I use to think that ok, these are the same things we did in grade 10 but as we go the long way in the long run, things got a little bit tougher. But in my class especially I love it because there are guys that can say, "sir, I have another way to get that." Then you kind of think that if he can get another way, why shouldn't it? And a lot of practice, and ... if you put a lot of concentration in the work you can actually get the way of what is it all about. It is just the way of finding formulae and understanding from where you started and how you will end and everything would be fine.
17	I	Ok! Do you feel you were mathematically well equipped from grade 10?
18	L ₅	Mm, grade 10 ..., I don't think so. Because I have a teacher who more is into jokes but he is a good mathematics teacher that I am sure of. The books that we used, they were just not too good for the mathematics that I experience now. I believe that if we were having more equipped books that have a lot of information and a little bit easy to explain, I think that could have made mathematics easier in grade 11.
19	I	Which part of grade 10 mathematics you think needed more attention?

20	L ₅	I think, the one that was neglected ... is the one where you can talk about angles, to find angles. And the other one ... mom ... ja! Factorising I didn't understand that very well in grade 10 mathematics.
21	I	Generally how was the test?
22	L ₅	For yesterday, I think it was really relaxing! Because the last time we wrote this, I was like, 'Oh, my God!' I was so stressed out I thought if I fail this test it would mean that I am like I won't do mathematics or anything like that. But for yesterday you made us more comfortable and relaxed and you told us to take our time and it does not have anything to do with our study. And I went through, I did... mathematics with time, and more relaxed, I was able to think through and do some of the stuff easily. But the others were kind of tough and that is where you get a challenge and everything.
23	I	Which part was more challenging in that test? You can page through your script.
24	L ₅	I won't forget about it, because I was really looking at it! Mmm... Question 11, I don't think it was tough.
25	I	Can you please read it aloud?
26	L ₅	Mmh... Expand and simplify $3x(x - 3) + 2x(2x + 1)$ equals to. There I look at the question and I understand that I have to expand what I am given...
27	I	What do you mean by the term 'expand'?
28	L ₅	I think like, you expand it, like you double it or something...
29	I	Mmh! Or you remove brackets?
30	L ₅	You have to remove the brackets? So then, if I didn't understand that, then I have to do that and the whole thing would be wrong. I didn't do it at all. I was looking at it and I thought if I do something that is not it, I would think of myself as being dumb.
31	I	Oh, no!
32	L ₅	I just hate it... (Inaudible).
33	I	But trying sometimes or most of the time, you will get it.
34	L ₅	My teacher says trying is better than nothing.
35	I	Ok. Anything else which you think was more challenging?

36	L ₅	Not really, the whole paper just needed concentration and understanding because if you don't understand anything I don't think you will have a point where to start... (Inaudible).
37	I	Mmm... I am interested in your working to question 5; can you explain how do you reach to that answer?
38	L ₅	Question 5, I am not so sure, if that is the right answer! But I thought about it that if I have to make x the subject, it means it would be x is equal to and the rest. So I thought if x was out of a , then I had to put $a \dots$ to put the rest in brackets and work up b and work up a , that is when I make x the subject. To me means you are now looking for x , that way you have to have a alone and to make it fair, you have to put c and b in brackets, mmm.... When to... mmm... aaa... crosses the equal signs it becomes a negative, aaa... when b crosses equal sign it, it will be negative b . Then c minus b outside the brackets and work from there.
39	I	Why b has to become negative?
40	L ₅	Cause if b has to go on this side crossing the equal sign, then if it was positive change to negative.
41	I	That is interesting! Mmm... I want us also to look at ... another question ... mmm ... ok, question 18!
42	L ₅	<p>There are 9 learners in a group, 6 are girls. What is the probability of picking a learner in a group who is a boy? Ok! I came across this question once, but guess I did something wrong of not asking what it really mean by probability. I think if ... I chose the ratio 6:3 because I worked it out, if you add 6 and 3 you get 9. That would be the sum of the ratio and then you get $\frac{6}{9}$ multiplied by the number of the learners then you get 6 which is already stated that they are girls.</p> <p>Then I wanted to find out, I took $\frac{3}{9} \times 9$ pupils and then I got 3 boys. That made sense, If you add 6 to 3 that will give you 9 learners. But I was not so sure, 'cause there might be the right answer which is three, which also make sense, but I thought of taking 6:3 and also show my work on how I got the whole point.</p>
43	I	If you have no clue of what is probability, how do you get to that answer?
44	L ₅	I heard of it already but I never really asked what I am supposed to do when I am asked such a question.
45	I	If you were a teacher of if you are to give advice to a mathematics teacher, what advice would you give?

46	L ₅	<p>I think that mathematics teacher should not so soft-hearted, such that they are not too tough. They do not come in the class with that face that where is your homework? And should not just only demand on homework, but they should demand on how actually a person work it out that homework.</p> <p>They should be those teachers who like explaining and if there is a question, they should go really deep into it. They just don't make you ask a question and then they give you another question in mind. I think mathematics teachers should be those that can really work with learners and have a good interaction with learners and accept co-operation in class. And they should not let learners make noise as people will loose concentration and stuff. They should be able to like teach and explain the whole way and just not to make things tougher.</p>
47	I	Thank you for your time!

APPENDIX E: PERMISSION LETTERS



REPUBLIC OF NAMIBIA
MINISTRY OF EDUCATION
OSHANA REGION

Tel: (065) 229800
Fax: (065) 229832

906 Sam Nujoma Rd
Private Bag 5518
Oshakati

Enquiries: Mrs. Helena Amuthitu
Ref: 10/3/4/1

Ms. Albertina N. Mwandingi
Gabriel Taapopi S.S.
Oshakati Circuit

4 April 2008

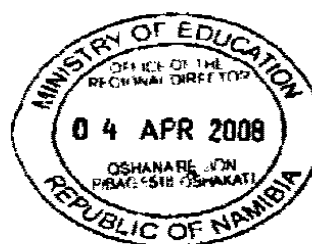
Dear MS. Mwandingi

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. Your letter dated 19 March 2008 is hereby acknowledged.
2. Please be informed that permission is granted to conduct research as requested. *However, the permission is only valid if it can be done as per condition as set out in the letter from the Permanent Secretary.*
3. This office is therefore waiting for the schedule and sample of questionnaire.

Thank you

7. *[Signature]* of 04.08.
MRS. DUTTE N. SHINYEMBA
REGIONAL DIRECTOR: OSHANA



All correspondence must be addressed to the Regional Director



REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION

PROGRAMMES AND QUALITY ASSURANCE

Tel: 264 61 2933200
Fax: 264 61 2933922
E-mail: mshimho@mec.gov.na
Enquiries: MN Shimhopileni

Private Bag 13186
Windhoek
NAMIBIA
9 April 2008

File: 11/1/1

Albertina Mwandingi
P. O. Box 3211
Ongwediva
Namibia

Dear Madam

SUBJECT: PERMISSION TO DO EDUCATIONAL RESEARCH

Your letter requesting permission to do research as part of the post-graduate studies for a Masters of Education Degree, through the Rhodes University, Grahamstown, South Africa, has reference.

Kindly be informed that the Ministry of Education recognises your effort and the possible contribution your research initiative can make towards successful curriculum implementation for education in a broader sense.

This letter grants you permission to do the required work in terms of consultations, interviews and other related interactions at both school and regional office levels.

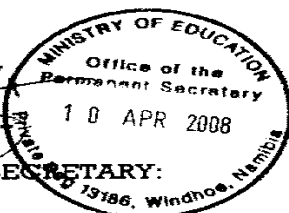
Kindly note that the Ministry of Education would expect from you to deposit copies of you published work in the respective libraries and resource centres. Also ensure that your research activities do not interfere with normal school programmes.

Best wishes for success in your academic endeavour.

Yours faithfully

.....
FV Angama

PERMANENT SECRETARY:



APPENDIX F: CONSENT FORM
CONSENT FORM

I hereby agree to participate in an in-depth interview with Albertina N. Mwandingi. I understand that she will be enquiring about my perceptions, beliefs and experiences towards mathematics and gains insight into the learning strategies/difficulties I use/have in learning Grade 11 mathematics.

I also understand that my confidentiality and anonymity will be considered after the interview and I will be consulted to verify the interview transcript and data.

Signed Date