An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making

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by

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ABSTRACT

The purpose of this Namibian case study was to investigate how different types of gestures are used to support the construction of mathematical meaning making in teaching and learning. Gestures of three selected Grade 8 mathematics teachers were observed and analysed. This study was intended to answer the following research question: how do selected Grade 8 mathematics teachers use gestures as visualisation tools to support mathematical meaning making? The study was framed by an enactivist perspective and the research was oriented in the interpretive paradigm. Data were collected through video-recorded observations of three selected teachers and through stimulus recall interviews. In order to generate rich data and support validity, five lessons per selected teacher were video recorded.

The study found that the participating teachers incorporated a variety of mathematical gestures into their lessons in order to support and provide mathematical meaning. Further, this study found that gestures facilitated meaning making in mathematics. The findings in the study suggest a need for mathematics teachers to be trained in using gestures appropriately to communicate mathematically in their lessons.

In addition, this study discovered a new type of gesture – the overlapping gesture in addition to McNeill's (1992) types, namely: pointing gestures, metaphor gestures, beating gestures and iconic gestures. This case study also showed that the more experienced the teachers are, the more mathematical gestures they produce during their mathematics lessons.

Keywords: gestures, meaning-making, visualisation, enactivism.

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DEDICATION

I would like to dedicate my work to my lovely, caring, supportive and inspiring fiancé, Maria Tutaleni Konstantin. To my sons David Tuhafeni Haipinge and Daniel Tuyoleni Haipinge – continue where I have ended, my sons, to ensure the DTH legacy lives on.

DECLARATION OF ORIGINALITY

I, David Tuhafeni Haipinge, student number 19H2096, declare that this thesis entitled *"An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making"*, is my own work and it has not been submitted for a degree or examination at any other university. Where I have drawn on ideas from other sources, I have fully acknowledged and referenced these in accordance with Rhodes University Education Department referencing guide.

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David T Haipinge 18 May 2021

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LIST OF ABBREVIATIONS AND/OR ACRONYMS

ACE	Advanced Certificate in Education
B	Beating gestures
BEd	Bachelor of Education
BETD	Basic Education Teacher Diploma
BODMAS	Bracket Of Division Multiplication Addition Substraction
I	Iconic gestures
IC	Interpersonal and Intrapersonal Communication
JA	Joint-attention
M	Metaphoric gestures
P	Pointing gestures
T1	Teacher 1 (Mr Kuutondowa)
T2	Teacher 2 (Ms Kambuta)
T3	Teacher 3 (Mr Katamba)
TA	Transformation of activities
VCD	Vocabulary and content development

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

The purpose of this first chapter is to introduce the reader to the background and goals of the research, which focuses on the use of gestures by selected Grade 8 mathematics teachers as visualisation tools, to create mathematical meaning. I describe the background of the study based on growing research and recognition of the importance of gestures in school. I also briefly indicate the theoretical underpinnings, research methodology and significance of the research. Lastly, the thesis outline is provided.

1.2 CONTEXT AND BACKGROUND OF THE STUDY

In recent years, there has been a growing research interest in the use and significance of gestures in both communication and construction of mathematical meanings and ideas. Several studies analysing gestures have been conducted. Rosborough (2010), for example, investigated gestures as a mediation tool for making meaning in the learning and teaching of a second language. Rosborough (2010) determined that gestures are used to direct attention, build inter- and intrapersonal communication in relation to content development and transform practice. Alibali and Nathan (2012) researched how the body is involved in thinking and communicating mathematical ideas. They confirmed that some aspects of mathematical thinking are embodied (Alibali & Nathan, 2012). In addition, Weinberg, Fukawa-Connelly and Wiesner (2015) found that gestures can address the complex ways in which mathematical meaning, both specific and general, are expressed.

My case study builds on a recent study by Namakalu (2018), conducted in Namibia with selected primary school mathematics teachers. The study described various types of gestures used by selected primary school teachers. Namakalu (2018) observed gestures in selected Grades 0 - 3 mathematics classrooms and concluded that gestures make lessons interesting, encourage the active participation of learners and can assist in explaining mathematical concepts. Her study focused on the role of the gestures, but she did not explore in any depth how mathematical meaning is communicated with gestures.

My study thus aimed to analyse how different types of gestures are used by selected Grade 8 teachers to enhance meaning making in school mathematics. This study sought to gain deeper insight into how gestures can be used in teaching mathematics to develop mathematical meaning. I argue in this study that the use of gestures can form a powerful mediation tool to communicate mathematically with learners. This does not only apply to learners who do not have a good language command and those who are hard of hearing, but to all learners.

1.3 RESEARCH GOALS AND QUESTIONS

1.3.1 **Research goals and significance of the study**

This study was an attempt to explore how different types of gestures used by mathematics teachers illustrate mathematical ideas and concepts in their teaching. This study contributed towards an improved understanding of the link between gestures, meaning making in mathematics and visualisation. It is also hoped that it will create awareness among mathematics teachers, researchers, policy makers and curriculum designers about the roles and impact of gestures in mathematics education. To achieve this goal, the research was guided by the following objective: to analyse selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making.

1.3.2 Research question

This study sought to answer the following question:

How do selected Grade 8 mathematics teachers use gestures as visualisation tools to support mathematical meaning-making?

1.4 THEORETICAL PERSPECTIVE

This study was underpinned by enactivism. According to Proulx (2013), enactivism "is an encompassing term given to a theory of cognition that views human knowledge and meaning-making as processes understood and theorized from a biological and evolutionary standpoint" (p. 5). Enactivism claims that human cognition does not occur in minds or brains only, but it brings together action, knowledge and identity so that there is a conflation of doing, knowing, and being (Davis, Sumara & Kieren, 1996). The biological structure (body, brain and heart) of an individual makes sense of the world in a holistic manner by a process of co-emerging all the sub-structures listed above (Maturana & Varela, 1992). In the context of my study, the teacher's act of gesturing is an integral and intertwined component of her body, heart, and mind, and of course her environment. In my analysis of teachers' gestures I was thus mindful of interpreting the gesture in the context of all of these elements. A gesture is therefore a manifestation of the mind, body, and heart in the context of the environment in which it takes place.

Similarly, a learner's interpretation and meaning making of a teacher's gesture occurs through being involved in observing the teacher's actions (gestures) in relation to her body, mind, heart, and the environment. Enactivism recognises the whole biological structure of an organism and the environment (Maturana & Varela, 1992) in relation to meaning making. In my study, the environment is the mathematics classrooms while teachers and learners are the organisms interacting with each other through gestures in order to communicate mathematical ideas.

In my case gestures are used specifically to aid the teachers in expressing mathematical ideas to learners. Simultaneously, the learners are observing the teachers' gestures, and these gestures are aiding learners to make mathematical meaning. The teachers' entire body is involved in thinking and speaking about the mathematical ideas by gesturing in the classroom (Alibali & Nathan, 2012).

1.5 RESEARCH METHODOLOGY

1.5.1 Research orientation

This study, situated in an interpretive research paradigm (Walsham, 2006), aimed to capture and interrogate the gesturing practices of selected teachers and their understandings of how they teach, using these gestures. Walsham (2006) highlighted that interpretive research involves capturing social reality as viewed and interpreted by the individual participants, according to their own ideological positions. In order to maintain the focus on the phenomenon under investigation, researchers in this paradigm view the world through the perceptions and experiences of the participants (Thanh, Thi & Thanh, 2015). In my study, I thus intended to gain a deep understanding of not only how selected Grade 8 mathematics teachers use gestures as visualisation tools to support mathematical meaning, but also how they themselves view their use of gestures in the classroom. In order to understand the participants' gestures and their interpretations of these gestures and the world around them, I observed and interviewed the participants and interpreted their actions and remarks about using gestures.

1.5.2 Methodology

This research is a qualitative case study that involves interactions with three selected Grade 8 mathematics teachers. The unit of analysis of this research is the teachers' practices, focusing on the use of gestures as visualisation tools to support mathematical meaning. According to Hamilton and Corbett-Whittier (2012), a case study "[i]s a way of framing a particular bounded unit, providing guiding principles for the research design, process, and communication" (p. 12). A case study is an appropriate methodology to use where the researcher wishes to understand in some depth the participants' views and experiences of the problem under investigation. The data for my unit of analysis are my observations of five video-recorded lessons per teacher, and one-on-one stimulus recall interviews.

1.6 STRUCTURE OF THE THESIS

This research study is structured as follows:

1.6.1 Chapter Two (Literature review)

This chapter discusses and reviews the literature pertinent to the study. This chapter contains four major sections, namely: (1) Gestures; (2) Visualisation; (3) Meaning making and (4) Enactivism – which is the theoretical underpinning of the study.

1.6.2 Chapter Three (Methodology)

This chapter focuses on the research methodology used to answer the research question. This chapter starts with the research orientation followed by the research methodology, research goal and research question, research design, research techniques, data analysis, validity and reliability, and it ends with ethics.

1.6.3 Chapter Four (Data analysis and discussions)

This chapter presents and discusses the findings of the research project drawn from fifteen lesson observations and stimulus-recall interviews. I firstly analysed the teachers' lessons and extracted all the mathematical gestures that the teachers used in their lessons. Each lesson observation was accompanied by an interview session.

1.6.4 Chapter Five (Conclusion and recommendations)

This chapter concludes the study by presenting the summary of findings, significance of the study and limitations of the study. It also makes some recommendations for further research. Finally, it ends with my personal reflections on my research journey.

1.7 CHAPTER SUMMARY

In this chapter, the context of the study, the research goal and research questions were discussed. In addition, the thesis outline was highlighted with the intention of helping readers to sail through the study. In the next chapter, relevant literature informing this study is reviewed and the theoretical framework underpinning the study is discussed.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

As a mathematics teacher for over nine years, there were times that I have unconsciously enacted multiple actions, like gestures, during my lessons. The use of these gestures support my verbal speech and often provides meaning to the mathematical ideas I wish to communicate. This study was thus aimed to analyse how these gestures, enacted by selected mathematic teachers, provide mathematical meanings to enhance the learning of mathematical knowledge. To achieve the objectives of the study, relevant literature on gestures, visualisation, meaning making and enactivism has been reviewed in this chapter. A literature review includes important readings outlined and combined in a specific way for the benefit of a study (Fink, 2015). Bertram and Christiansen (2015) add that a literature review displays significant research done in the field being investigated and explores how these studies are linked to each other.

This chapter contains four major sections, namely: (1) Gestures; (2) Visualisation; (3) Meaning making and (4) Enactivism – which is the theoretical underpinning of the study. The first section includes a history of gestures, definitions of gestures; gestures and speech as an integrative system; types of gestures; roles and functions of gestures; stereotypes of gestures; criticisms of gestures and lastly, difficulties around gestures. Furthermore, in the second section, visualisation is covered in some details viz: the definition of visualisation; the role of visualisation in the teaching and learning of mathematics; and visualisation and gestures, ending with difficulties with visualisation. In the third section, the concept of 'meaning making' is discussed, focusing on the definitions of meaning making as well as meaningmaking and gestures. The last section covers the concept of enactivism which is the theory underpinning this study. This section includes the definition of the following concepts: enactivism, embodied cognition, gestures as embodied actions, and co-emergence, ending with co-emergent processes and gestures. The chapter concludes with a discussion on the use of gestures as a visualisation tool to support mathematical meaning-making.

2.2 GESTURES

2.2.1 History of Gestures

People communicate in different ways: either verbally or non-verbally. Verbal communication is the use of sounds and words to express oneself, while non-verbal communication is the use of bodily actions to communicate (Hewes, 1973). Kendon (2004) adds that the use of bodily actions (hands, face or other parts of the body) is regarded as gestures. The term 'gesture' comes from the Latin word '*gestura*' which means the mode of action, manner and bearing (Johannesson, 1944). Bearing refers to an act of standing or moving (Sander & Louise, 2002). The study of gestures has been in existence since the 5th century, BC (Allan, 2013). For example, studies have been conducted on how people greet each other by gesturing and shaking hands (ibid.). In addition, Allan (2013) stated that in the 17th century, during wedding ceremonies the bride and groom used gestures to signify the sealing of a sacred and legal agreement.

In a quest to understand the use of gestures and their effectiveness in enhancing communication in general, and thus facilitating learning in particular, the subject of 'gestures' have been under investigation by many scholars (McNeill, 1992; Namakalu, 2018; Ntuli, 2012) since the 19th century. In 1992, David McNeill, a linguistic professor at the University of Chicago, conducted research on gestures and language. Based on the results of his study, it was concluded that gestures transform thoughts – which language cannot always express – to form visible ideas (McNeill, 1992). Ntuli (2012) from South Africa explored how gestures or non-verbal communication can cause misunderstandings among South African cultures. Ntuli (2012) highlighted that there was little social interaction between South Africans during the colonial era. For that reason, she found that it was difficult for most cultures to understand each others' gestures, because gestures cannot be separated from a person's culture and socialisation (ibid). Namakalu (2018) from Namibia investigated the types of gestures used by selected primary school teachers in Namibia. She observed gestures in selected Grade 0 - 3 mathematics classrooms and concluded that gestures make lessons interesting and encourage the active participation of learners. She also indicated that gestures help in explaining mathematical concepts. The focus of her study was confined to describing the roles of gestures. There is still a knowledge gap in establishing a deeper understanding of how mathematical meaning is communicated with and through gestures.

To further Namakalu's (2018) work, this study aims to analyse how selected Grade 8 teachers use different types of gestures to enhance meaning making in school mathematics. This study seeks to gain a deeper insight into how gestures can be used in teaching mathematics to develop mathematical meaning. It is argued in this study that the use of gestures can form a powerful mediation tool to communicate mathematically with learners. This does not only apply to learners with a bad language command or those who have a hearing disability, but it can benefit all learners. This study does not deal with everyday communication gestures. It is rather focused on gestures that selected teachers use to teach mathematical ideas. This study was thus classroom bound. As the teaching gestures that were observed were directed at learners, they had to be clear and unambiguous for learners to understand and interpret correctly.

2.2.2 Definition of gestures

Scholars are yet to find consensus on the definition of gestures. According to Cook, Friedman, Duggan, Cui and Popescu (2017) gesturing is a process of communication whereby people use "non-verbal behaviours, including eye gaze and prosody along with face, lips and body movements" (p. 518). Gestures are described as all types of bodily, non-vocal communicative acts, particularly hand motions, body postures and facial expressions (Kendon, 2004). In addition, Sfard (2009) stated that gestures include any "body movement fulfilling a communicational function" (p. 53). Moving of arms, hands and other parts of the body to communicate mathematical meaning to the learners in one way or another by the teacher in the classroom is classically referred to as gestures. Gestures are universal features of communication in both children and adults, and often occur simultaneously with speech (Kirk, Pine & Ryder, 2011). Lastly, De Freitas and Sinclair (2012) argued that when gesturing, the body forms part of a complex network by which the social and the content (in this case mathematical content) are glued.

De Freitas and Sinclair's (2012) definition of gestures resonates well with the purpose of my study since the social unit in my study refers to the interaction between mathematics teachers and learners in the classroom, and the content is Grade 8 mathematics. In my study, gestures are observed when participating teachers explain and give mathematics instructions to the learners using actions. As Flevares and Perry (2001) emphasise, it does not make sense for teachers to give all the important information verbally. Hence, mathematics teachers use representations (such as gestures) to “accompany important spoken terms and to respond to student confusion” (ibid., p. 329).

2.2.3 Gestures and speech as an integrative system

When people talk, the use of gestures becomes an integral part of the speech. For example, Alibali and Nathan (2007) observed that mathematics teachers routinely produce gestures along with speech when teaching mathematics concepts. McNeill (1992) asserted that gesture and speech must work together to help put across the message clearly to the target audience.

A few authors such as Castellon and Enyedy (2008), Lin (2017) and Radford (2003), all focused their research on natural gestures linked with speech. Radford (2003) explored the relationship between gestures, speech, and the developing of symbols, by studying the process of how students used body gestures when discussing mathematical matters. He discovered that there is a link between speech and gestures, for example, when students perform pre-symbol actions/gestures of a general mathematical idea before verbalising it.

In a different study, Castellon and Enyedy (2008) investigated the gestures and speech of teachers as a communicative way to convey mathematical thoughts to the learners. The results of their study show that the use of both gestures and speech by a teacher helped with explaining mathematical concepts to students. Furthermore, Castellon and Enyedy (2008) indicated that the use of both gestures and verbal communication resolves multiple meanings, elicits students’ justification of their thinking, and, most importantly, advances the mathematical lesson. A study by Lin (2017) which investigated the relationship between the use of gestures and English proficiency level as well as cultural background, established that both the English proficiency level and cultural background have no influence on the gesture-speech relationship. In her study, Lin (2017) considered Taiwanese, Indonesian and Indian speakers. Thus, in this investigation, no consideration was given to the English proficiency level or culture/language background of the teachers when choosing the participants.

An interesting observation by De Ruiter (2000) was that “people do not only use gestures to represent something for others, but also when there is nobody else to see the gestures, for example, when they are talking on the phone” (p. 290). This highlights that gesture and speech are indeed intertwined. In this study, I observed how the mathematics teachers use gestures along with speech to create mathematical meaning.

2.3 TYPES OF GESTURES

Several scholars such as Edwards (2003), Kendon (1997) and McNeill (1992), have identified different types of gestures used in teaching and learning. This study focuses in particular on gestures that selected mathematics teachers' use in their teaching to communicate mathematical meanings and ideas. McNeill (1992) categorised gestures into four main types, viz. pointing, metaphoric, iconic and beat gestures. These are discussed as follows:

Pointing gestures: these gestures are used as "entities and actions in space vis-a-vis a reference point" (McNeill, 2006, p. 40). Pointing gestures can be either real, to point at a definite object being deliberated; or nonconcrete, such as pointing at a piece of writing on the chalkboard. A further example of a pointing gesture is the teacher pointing out specific angle properties on a diagram that he/she has drawn on the chalkboard.

Metaphoric gestures: these gestures represent ideas or concepts that do not have a physical form. They are used to describe a shape, reinforce, affirm and explain ideas such as waving of hands, to for example, indicate the size of the mathematical shape or a specific shape (McNeill, 2006). A teacher could use metaphoric gestures to indicate a clockwise or anti-clockwise rotation with her fingers or hands instead of using the board to draw rotation directions. The teacher may also use arms to gesture parallel lines accompanying the verbal explanation of parallel lines.

Iconic gestures: these gestures resemble the semantic content of speech and illustrate what is being said. Iconic gestures are useful because they complement what the speaker is saying. They portray meaning with the movements of the hands. For instance, a teacher uses gestures to tell and describe features of height to the learners by illustrating different heights, so that learners can use these features to support their understanding of height (Shein, 2012) for example, by moving hands apart from each other to model the heights.

Beat gestures: These gestures emphasize a rhythmic or time-based aspect of the speech. Beats can be "mere flicks of the hand(s) up and down or back and forth, zeroing in rhythmically on the prosodic peaks of speech" (McNeill, 2006, p. 23). He added that beat gestures are speech related due to the movement people make when speaking. Nevertheless, beat gestures also have "discourse functionality, signalling the temporal locus of something that the speaker feels to be important with respect to the larger context" (McNeill, 2006, p. 23). Beat gestures have been found to occur often as iconic gestures during "successful resolution of a tip-of-the-tongue situation" (Llanes-Coromina, Vilà-Giménez, Kushch, Borràs-Comes & Prieto, 2018, p. 168). One example of a beat or rhythmic gesture is the regular tapping of a finger on the table, while thinking. Another example would be the clapping of hands while counting out a sequence of numbers.

This study recognises that it is difficult, at times to classify gestures into only one discrete category because gestures often overlap – i.e. a pointing gesture is used as a beat gesture or an iconic gesture. Therefore, during my data collection and analysis, this has been taken into consideration and thus "overlapping gestures" have been included as an additional category to McNeill's four types of gestures.

2.4 ROLES AND FUNCTIONS OF GESTURES

In recent years, there has been a growing research interest in the use and significance of gestures in both communication and the construction of mathematical meanings and ideas. Several studies analysing gestures, discussed in the subsections below, have added to the body of knowledge on gesture roles and functions.

2.4.1 Rosborough's Functions of gestures

Rosborough (2010) investigated the use of gestures as meaning-making tools in an English second language context. In the coding of his data, Rosborough also considered the four gestures as categorised by McNeill (1992), i.e. pointing, metaphoric, iconic and beat. His study, however, focused specifically on the functions of the gestures. He found that gestures facilitated meaning making in the English second language by joint-attention, interpersonal and intrapersonal communication, vocabulary and content development and transformation of activities. These functions for gestures are discussed in detail below.

a) Joint-attention or shared attention

Joint-attention takes place between two gesturing individuals who are communicating with each other. Scaife and Bruner (1975) indicated that "joint-attention is important particularly for language development including comprehension, construction and symbol learning" (p. 265). It can thus be argued that in a mathematics classroom, if learners are not paying attention to what the teacher is explaining both verbally and through gesturing, the use of gesture becomes meaningless. In this study, the role of joint-attention will be analysed for each of McNeill's type of gestures when it comes to the mathematical learning process in classrooms.

b) **Interpersonal and intrapersonal communication**

Interpersonal communication "takes place between people; whereby people communicate with each other for reasons such as to explain, to teach, to inquire, and to inform" (Rogers, 1951. p. 18). Rogers (1951) found that "interpersonal communication is a loop process with four basic elements, i.e. sender, message, medium and receiver" (p. 19). In my study, the teacher is the sender of the message by gesturing (the medium) and in return, the learners receive the message (the mathematical concepts being taught).

On the other hand, as the name implies, intrapersonal communication occurs "within individuals" (Rogers, 1951 p. 18) and it is used for analysing situations, clarifying concepts and reflections. Intrapersonal communication is self-centred because the sender and receiver are the same person. The message is made up of one's own thoughts and feelings. Honeycutt, Zagacki and Edwards (1987) explain that "the channel of intrapersonal communication is your mind" (p. 1). The mind, in turn, processes what one are thinking and feeling. "There is a feedback in the sense that you talk to yourself or discard certain ideas and replace them with others" (Honeycutt et al., 1987 p. 1). An example of intrapersonal communication could be a teacher meditating without writing (Burger, Lee & Rust, 2018).

c) **Vocabulary and content development**

To teach and develop vocabulary, or in my case, mathematical content, teachers often model the content or words by using gestures. This provides content clues to the learners on how to tackle a mathematical problem. Through a simple movement that mimics the concept or meaning of the word it becomes easier for the learners to understand and internalise content and vocabulary. For example, a "teacher uses gestures to indicate or request objects before they produce words for these referents" (Nathan & Alibali, 2012, p. 248).

d) **Transformation of activities**

Teachers who transform and add value to mathematical ideas often make use of gestures when communicating. The model in Figure 2.1 illustrates the steps involved in a typical transformation process. The transformation process involves three stages: input, process and output. For example, the input could be a clue provided through gesturing by the teacher, on how to tackle a mathematic question. In the second stage, i.e. the process, the message through gesturing is received and processed (possibly through some process of visualisation) by the learners. Finally, as an output for the transformation process, the learners provide answers to the posed question. This can also be considered as a meaning-making process that involves learners making connections between gestures and mathematical concepts.

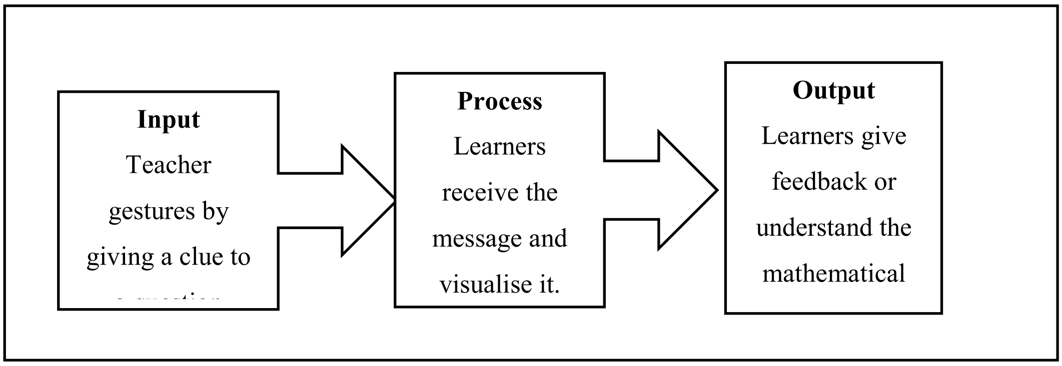

Figure 2. 1: The transformation process of activities brought about by gesturing

In Table 2.1, a synthesis of McNeill's (1992) types of gestures and Rosborough's (2010) functions of gestures is discussed. This forms part of my analytical framework. The new category of gesture types – overlapping gestures – has been added to the types of gestures already discussed in Section 2.3. In the results chapter, the types of gestures proposed by McNeill (1992) that were produced by participants, and their functions towards enhancing mathematical meaning, as suggested by Rosborough (2010), are discussed.

Table 2.1: Types of gestures and Rosborough's (2010) functions of gestures

<table>
<tr><th>Four McNeill (1992) gestures including overlapping gestures</th><th>Rosborough's (2010) functions of gestures for meaning making</th></tr>
<tr><td>Pointing gestures
Gestures that point to, or indicate something.</td><td rowspan="5">Joint-attention or shared attention (JA)
Joint-attention takes place between two gesturing individuals whereby one individual communicates with another person and vice versa. It can be associated with a situation where the teacher and learners are looking at one point at the board (Kaplan & Hafner, 2006) and deliberating about it.
Interpersonal & intrapersonal communication (IC)
Interpersonal – teacher talking to the learners accompanied by actions/gestures.
Intrapersonal – teacher talking to him/herself and gesturing, e.g. head scratching while thinking of the answer. Head scratching can be used to ask the learners to engage in deep thinking.

Vocabulary & content development (VCD)
Gestures reinforcing content. (e.g. making the point of a rhombus using hands and fingers) before the teacher produces a verbal label for the rhombus object.

Transformation of activities (TA)
Converting of gestures into mathematical context. Connection of gestures to real life mathematics e.g. a teacher makes a co-interior angle with arms, and the learners will connect it to mathematical context of the ‘U’ angle.</td></tr>
<tr><td>Beating gestures
“… are motions that help emphasize or keep a rhythm in a person’s speech” (McNeill, 2006, p. 23).</td></tr>
<tr><td>Metaphoric gestures
“Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about” (McNeill, 2006, p. 23).</td></tr>
<tr><td>Iconic gestures
Imitate something by physically looking like that thing.</td></tr>
<tr><td>Overlapping gestures
These are two or three gestures that overlap and are not discrete.</td></tr>
</table>

2.4.2 Gestures as communication tools

As has already been explained in Section 2.2.3, gestures and speech are intertwined and it can thus be stated that for effective communication to take place, gesturing must be considered. Several studies have indicated that the use of gestures in mathematical classrooms contribute to effective communication of the concepts. Amalancei (2014) stated, “no matter how many meanings they might have, words cannot always transmit everything or enough to reach the target for which they are produced” (p. 903). In a different study, Valenzeno, Alibali and Klatzky (2003) said that teachers’ gestures play an important role in helping students to understand their instructions if gestures are used along with speech. Therefore, it is important for teachers to integrate gestures in their teaching in order to communicate clear instructions to the learners. It is thus important that when explaining certain concepts to learners, a teacher make multiple gestures as a supplement to verbal communication. An important aspect to emphasise here is that the use of gesture is not necessarily a replacement of verbal communication, but rather a supplement. Amalancei (2014) added that gestures could, however, only be important communication tools if the receivers (learners in the classroom) understand their meanings. For whatever medium of communication being used, it is important that the message passed on to the receiver by the sender should be clear and understood. Contrary to this, in the case of mathematics teaching, learning does not occur if the used gestures are not intelligible to the learners. This study examines the use of gestures in classrooms, as communication tools, to enhance mathematical learning.

2.4.3 Gestures as mathematical tools

In the previous section, it was emphasised that gestures are an integral part of a communication as they supplement speech to ensure a good grasp of the message by the intended audiences. Gestures can thus be used as teaching aids in mathematics classrooms. Over the years, several researchers have been working to find the link between gestures and the teaching of mathematics. For instance, Flevares and Perry (2001) investigated the use of gestures in Grade 1 mathematics lessons. Their results show that gestures improve mathematical communication between teachers and learners. In other words, teachers' gestures might accompany some information about mathematics concepts that words/speech cannot convey, such as a demonstration of a curved graph and line of symmetry. Furthermore, Edward's (2005) study on the role of gestures in mathematics teaching stated that gestures are "seen as important bridges between imagery and speech and a nexus bringing together actions, imagery, memory speech and mathematical problem solving" (ibid, p. 135). This is further discussed in Section 2.8.3 where I created a model based on Goldin-Meadow's and Susan (2015) visualisation tools and gestures.

In addition, a study by Francaviglia and Servidio (2011) that was conducted at a primary school in Italy, investigated the relationship between gestures and mathematical problem solving. Their results suggest that gestures facilitate students' learning of mathematical thoughts and improve learners' cognitive strategies to solve mathematical problems.

Further to the discovery by Cook et al. (2017), Francaviglia and Servidio (2011) stated that gestures help learners to transfer and generalise their mathematical knowledge into real life problems. In their study, children viewed recorded videos of mathematics lessons under two conditions: lessons with gestures and lessons without gestures (Cook et al., 2017). The outcome was that the children who observed the lessons with gestures were able to understand the concepts and could thus solve mathematics problems more quickly and correctly. It can thus not be emphasised enough that gestures aid mathematics learning and that the integration of gestures in Namibian mathematics classrooms should be considered. This has the potential to improve mathematics academic performance in all grades, thus making the realisation of Namibia's vision 2030 possible – as many learners could then pursue science and technology courses that are necessary for industrialisation as per the Namibia Ministry of Education strategic plan (MoE, 2017).

2.4.4 Gestures as teaching tools

It has been emphasised in the preceding sections that gestures can be an effective communication tool. Teachers can make use of gestures as a teaching aid to enhance mathematics learning in their classrooms. The gestures that teachers produce are useful teaching aids because they "supplement the spoken terms to ensure that the learners understand the concepts being taught" (Flevares & Perry, 2001 p. 330). Chue, Lee and Tan (2015) state that gestures in the classroom show important information about "size, relative position and movement of particles" (p.1). Chues et al.'s (2015) investigation resonates well with my study with the only difference being that Chue et al.'s (2015) analysis was in science teaching while my investigation is in mathematics teaching.

2.4.5 Gestures as learning tools

As Amalancei (2014) explains, the use of gesture as a communication tool, can only be effective if the learners understand the used gestures. Various researchers have investigated how the use of gestures by teachers helps to enhance mathematical learning, i.e. how the learners benefit from the gesturing teachers. Valenzeno et al. (2003) found that teachers' gestures help students to grasp instructional discourse. Similarly, Goldin-Meadow and Ping (2008) found that the children given instruction through both speech and gestures tend to learn more than those given instruction only through speech. Additionally, "children who received instruction in both speech and gestures were more likely to explain how they solved problems" (ibid., p. 1).

Furthermore, Goldin-Meadow and Susan (2015) adds that gestures "are able to highlight components of an action that promote abstract learning" (p. 167). Abstract ideas or thoughts do not have a physical or concrete existence (Wu, Dale & Bethel, 1998). For example, in a classroom, a teacher may use her arms to show the direction of rotation, i.e. clockwise or anticlockwise. In general, clockwise and anticlockwise do not exist in physical form and can thus be considered abstract ideas. Finally, Yeo, Ledesma, Nathan, Alibali and Church (2017) add that gestures help learners make links between teachers' different representations of mathematics and mathematical thoughts.

2.4.6 Gestures lighten the cognition load

> *"Speakers move their hands when they talk; they gesture in all cultures and at all ages. Even congenitally blind individuals who have never seen anyone gesture move their hands when they talk suggesting that gesturing is a robust part of speaking"* (Goldin-Meadow & Susan 2014, p. 4).

There are times when teachers and learners encounter challenging mathematics problems and in the process of solving such problems the teachers and learners often produce multiple gestures as they think of the solutions. The gestures produced by the teachers and learners during a problem-solving process indicate that their bodies try to help their mind to think. This has been supported by Alibali and Nathan (2012) who state that "gestures are taken as evidence that the knowledge itself is embodied and the body is involved in thinking" (p. 247).

Moreover, Goldin-Meadow and Wagner (2005) examined whether gestures can lighten a person's cognitive load. This was done by giving the learners mathematics problems to answer under two conditions. The first condition was that learners answer the mathematical questions while their bodies were still, static or not moving. The second condition was that learners could move their bodies when solving the given mathematical problems. The results showed that when learners use gestures when solving mathematical problems, they tend to solve problems faster than when compared to when they are not gesturing.

2.4.7 Gestures pave the way for language development

Numerous studies have examined how gestures help with language development. For instance, Goodwyn, Acredolo and Brown (2000) evaluated how gestures helped children in their early development. They divided infants into two groups: one for parents who were trained to encourage their infants to use gestures; and the other for parents who were not trained to use gestures with infants. Goodwyn et al.'s (2000) results provide strong evidence that symbolic gestures do not hamper verbal development and may even facilitate it. For this reason, the infants from the trained group started talking before those infants from the non-intervention control group.

In addition, Iverson and Goldin-Meadow (2005) examined whether children's gestures are linked to their language development. Their result where positive because children first gesture before producing words (ibid.). Besides that, Iverson and Goldin-meadow (2005) observe that whenever there is a change in children's gestures, there is also a change in their language development. In my study, I focused on participants' actions during their lessons while they were explaining mathematical thoughts. In the next section, I will discuss stereotypes around gestures.

2.5 STEREOTYPES OF GESTURES

Although gestures appear to be universal in many cultures, people labelled the usage of gestures differently. For instance, Sekine et al. (2015) analysed 363 undergraduate students' opinions about gestures. These students came from France, Italy, Japan, the Netherlands and USA. The results show that some people believe that "Italian, Spanish, and American-English speakers gesture more frequently than any other language group" (Sekine et al., 2015, p. 91). It is also believed that each cultural group across the globe put weight on some features of gestures (ibid.). In their study, they also indicated that some people believed that "speakers in East Asia are not supposed to gesticulate, and it is generally believed that Italians gesticulate more than the British" (ibid., p. 91). In a different study, Kendon (2004) stated that some cultures are richer in gestures than others. In some countries, the use of gestures is even considered indecent (Heinrich & Stahl, 2017; Herzfeld, 2009). From my own experience, in some Namibian cultures, the use of gestures is not allowed when talking to elders as it is interpreted as having a lack of respect.

2.6 CRITICISM OF GESTURES

Even though the use of gestures plays a vital role in teaching, learning and communicating mathematical ideas, some studies have been rather critical about the use of gestures in teaching and learning. For example, Oh (2017) stated that gestures could cause disruptions in the classroom, particularly if the gestures seem inappropriate and cause people to feel embarrassed, leading to unnecessary laughter and scorn. Oh (2017) observes for example, how American-born Chinese students often get teased and laughed at every time they gesture in the classroom.

2.7 DIFFICULTIES AROUND GESTURES

In some cases, gestures can be difficult to understand because the gesturer can mean something different to the interpretation of the gesture by the person at the receiving end. For instance, a teacher produces a gesture of parallel lines, but the learners might think the teacher is trying to imitate either horizontal or vertical lines. In cases like this, there would be no learning taking place because the person at the receiving end of such a gesture might not be able to interpret the meaning behind such gestures. In the classroom observations of my study, for example, I might have initially misinterpreted some of the participants' gestures. The stimulus recall interviews were thus used to ask the participating teachers themselves about the meanings of the gestures that they used when teaching.

Furthermore, there are times we find a gesture mismatch in people's speech, when "the information conveyed in the speech is not matching the information conveyed in the gestures" (Church & Goldin-Meadow, 1986, p. 43). Ping et al. (2019) posit that individuals who produce gesture mismatches are regarded to have some cognitive instability. This eliminated any ambiguities that may have arisen in my analysis. I have synthesised Table 3.5 as my Analytical Tool 2 for the interviews.

2.8 VISUALISATION

2.8.1 Definition of visualisation

Different scholars define visualisation differently. For example, Zimmermann and Cunningham (1991) defined visualisation as a "process of forming mental images and using such images effectively for mathematical discovery and understanding" (p. 3). Thus, "to visualize is the ability to create a rich and mental image, which the individual can manipulate in the mind to rehearse different representations of mathematical ideas" (Goldin & Shteingold, 2001 p. 1). In this study, I have adapted Goldin and Shteingold's (2001) definition, because whenever a teacher gestures, learners create mental images in their minds of what the gestures are trying to communicate. These gestures may then remain in the learners' memory, because according to Yakovlev, Amit and Hochstein (2013), images are easier to remember than words.

2.8.2 The role of visualisation in the teaching and learning of mathematics

Some researchers say that words alone, without visual connotations cannot say everything intended (Amalancei, 2014; Valenzeno et al., 2003). Thus, in this section, I discuss the importance of integrating visual material into the teaching and learning of mathematics. Barwise and Etchemendy (1991) pointed out that "visual forms of representation can be important ... as legitimate elements of mathematical proofs" (p. 9). Moreover, Arcavi (2003) added that visualisation helps us to see the unseen. The "unseen refers to what we are unable to see because of the limitations of our visual hardware, e.g. because the object is too far or too small" (ibid., p. 216). In my study, mathematics teachers used gestures to support learners' visual limitations of, for example, a cube or cuboid drawn on a plane surface or a dice or handmade cuboid brought to the classroom. Such gestures enable learners to better see mathematical ideas and concepts.

Furthermore, Giaquinto (2007) argues that "visual means are much more than a mere aid to understanding and can be resources for discovery and justification, even proof" (p. 402). In addition, Hessler and Mersch (2009) add that visualisation could be used as a translator. In other words, the presence of the picture (visual) can explain what words cannot otherwise explain. Further, Konyalioğlu, Aksu and Şenel (2012) observed in their study that some mathematics teachers prefer to make use of visual aids when teaching absolute value because it has a constructive impact on the learners' learning. Thus, "visualization can also fulfil a heuristic function for solving problems in which case the used diagrams such as graphs and geometrical figures are intrinsically mathematical and are used for the modelling of real problems" (Duval, 2014, p. 159). By the same token, Natsheh and Karsenty (2014) state that the use of visual tasks in mathematics can promote and enhance conceptual understanding. Finally, Klerkx, Verbert and Duval (2014) say that in drawing and mapping work (e.g. graphs of function and geometry), mathematics teachers use visual learning "to enable users to control the process of flexibly navigating through information spaces of abstract data, for which there may be no inherent mapping to a space or a natural physical reality" (p. 1).

In a different study, Muhembo (2018) suggests that the "appropriate use of visualization helps learners to develop their mathematical conceptual understanding as it allows them to visually interpret and understand fundamental mathematics" (p. 2). He further claimed that visual tools play an important role in communicating mathematical ideas through diagrams, gestures, images, sketches or drawings. "Learning mathematics through visualization can be a powerful tool to explore mathematical problems and give meaning to mathematical concepts and relationships between them" (Muhembo, 2018, p. 2).

In this study, I acknowledge that modelling is also a form of visualisation, because modelling has the "ability to represent, transform, generate, communicate and document on visual information (Hershkowitz, Arcavi & Bruckheimer, 2001, p. 255). In addition, Blum and Ferri (2009) found that modelling can help in constructing a mathematics problem to become more meaningful for learners, viz. it helps students "to better understand the world, support[s] mathematics learning (motivation, concept formation, comprehension, retaining), contributes to develop various mathematical competencies and appropriate attitudes, and contribute[s] an adequate picture of mathematics" (p. 47).

2.8.3 Visualisation tools and gestures

Visualisation tools are objects such as graphs, charts, maps and gestures that make it easier for people to see and understand abstract ideas (Whitney, 2013). In this section, I focus on different ways in which gestures can be used as visualisation tools. For instance, Hecht, Vogt and Prinz, (2001) posit that gestures influence visual awareness in people. Alibali (2005) added, "Gestures are particularly good at expressing spatial and motor information" (p. 2). In my study, mathematics teachers asked learners to draw a circle, while drawing a circle in the air. In addition, Alibali and Nathan (2012) argue that "pointing gestures reveal (visualise) speakers' indexing of speech to the environment" (p. 247). In addition, Goldin-Meadow and Susan (2015) says that gestures serve as a bridge between abstract mathematical ideas and specific contexts. In this study, I adapted Goldin-Meadow and Susan (2015) use of gestures as a visual tool and synthesised a sketch on how gestures serve as a bridge to mathematical thought in Fig 2.2 below.

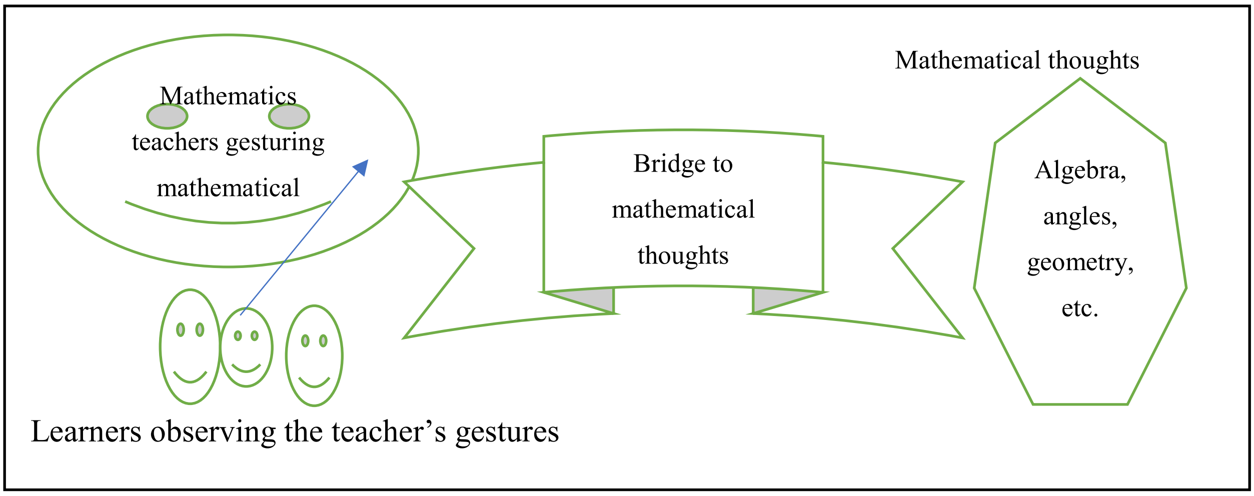

Figure 2. 2: Gestures as a bridge between thought and mathematics

Gestures as visualisation tools can be used in school. For examples "gesture points out objects in the immediate context and thus helps ground the words learners hear in the world they see" (Goldin-Meadow, Susan & Ping, 2008, p. 1). Moreover, Castellon and Enyedy (2008) argue that "a gesture is an important visual resource that can play a valuable role for English Language Learners (ELLs) in creating effective discourse practices and environments" (p. 1). Again, Goldin-Meadow and Susan (2015) adds that gestures can be used to represent the world that is not present now. In this study gestures represent the world that is absent from the classroom. In other words, absent and abstract concepts can be brought into the classroom with gestures. Finally, Ng and Sinclair (2013) suggest that "children use gestures as multi-modal resources to communicate temporal relationships about spatial transformations" (p. 360). For instance, children forming a cone with their hands.

Many authors as one of the readily available visual materials that can be used to teach mathematics view gesture. For instance, Castellon and Enyedy (2008) observed how teachers used gestures to convey and discuss mathematical concepts in algebra. In addition, Maschietto and Bussi (2009) used gestures, drawing and speech to construct visual pyramids in order to create mathematical meaning for the learners.

Different researchers view visualisation in varied ways. Hershkowitz, Arcavi and Bruckheimer (2001) for example, view visualisation "as the ability to represent, transform, generate, communicate, document and reflect on visual information" (p. 255). These six visualisation abilities resonate well with my study. Therefore, they are used to describe what happens in my mathematics classrooms during my observations. As the teachers represent mathematical ideas/concepts with parts of their bodies, the learners convert these movements or actions into something meaningful and create mental images of what is being represented by the teachers. Drawing from Hershkowitz et al.'s (2001) view, I created a flow diagram shown in Figure 2.3 to explain this visualisation process.

Represent (teacher's gestures),

↓

Transform (learners try to internalize the teacher's body movement),

↓

Generate (learners start to create the meaning of the teacher actions),

↓

Communicate (learners understand the meaning of the teacher's actions),

↓

Document (learners make connections to the mathematical problem)

↓

Reflect on visual information (learners validate of what they have visualized).

Figure 2. 3: Flow diagram showing the visual process when gestures are used.

Presmeg (2014) indicated that without gestures it is difficult "for a teacher to present a visual display in teaching a mathematical topic and expect learners to make the intended connections with the mathematical idea" (p. 1) and the teacher's actions. Presmeg (2014) recommends that teachers should include gestures as part of the visual display in their teaching. This resonates well with my study because I argue that visual displays in conjunction with gesturing helps learners to make connections and make mathematical meaning.

2.8.4 Difficulties with visualisation

Several studies have identified some difficulties with visualisation. For instance, Dreyfus (1991) stressed that it is important for both teachers and learners to be aware of difficulties that might arise due to improper or inappropriate use of visualisation, such as difficulties in reading graphs correctly, a lack of distinction between geometrical images and their visual presentation, and in the case of this study, a misunderstanding of gestures and their meanings. Furthermore, Arcavi (2003) proposed key groups of difficulties with visualisation and they are as follows: the first is "[v]isual settings imply that there are not always procedurally 'safe' routines to rely on (as may be the case with more formal symbolic approaches)" (p. 220); and the second is that the process of learning to understand and handle multiple representations is long-winded and tortuous for learners. This might happen in my study when learners are trying to understand mathematics teachers' gestures. Lastly, Konyalioğlu et al. (2012) state that "relying too much on visualization may prevent mathematical thinking due to the limiting effect of a single-case scenario represented by an image" (p. 615).

2.9 MEANING MAKING

2.9.1 Definition of meaning making

Meaning making is described as a process of "how people interpret, understand or make sense of life events, relationships" (Crowder, 1996, p. 173) and themselves. On the other hand, Klein (2017) add that meaning-making is the ability to help people (in this study, the learners) to make sense of life events (in this study, mathematical ideas). The sense that learners are making in my study is the connection between teachers' gestures and mathematical ideas. In my study, I observed how teachers use gestures in selected Grade 8 mathematics classrooms in order to help learners make connections/sense of mathematical ideas. Lastly, Braasch and Bråten (2017) define meaning-making as a process "of reading that engages readers in three different kinds of thinking: **comprehension, understanding** and **evaluation**" (p. 43). My intention is to transfer Braasch and Bråten's (2017) definition into a mathematical learning context. Drawing from Braasch and Bråten's (2017) process of meaning-making I have created a flow diagram shown in Figure 2.4 to illustrate the process that occurs as the teacher gestures in a mathematics classroom.

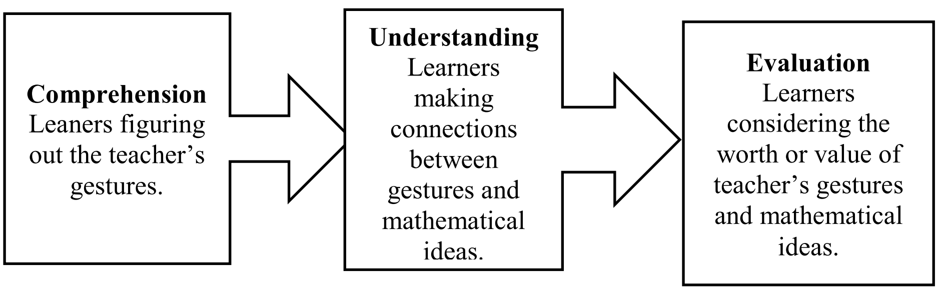

Figure 2. 4: The meaning-making process in mathematics

2.9.2 Meaning making and gestures

When teachers produce gestures to accompany their speech in the classroom, not all learners grasp the meaning of these gestures. Thus, some studies tried to make sense of those gestures produced by teachers in the classroom. For instance, Rosborough (2014) examined how English teachers make meaning using gestures. Rosborough's (2014) study was "responding to Thibault's (2011) call for understanding the language through whole-body sense making, aspects of gesture and body positioning" (p. 227). Moreover, Rosborough (2014) found that "gestures along with bodily positions and [inter]actions play a central role in this dyadic meaning-making experience" (p. 227). For this reason, in this study I synthesised Table 3.5 as the Analytical Tool 1 for the observation. Table 3.5 is the first of the analytical tools I devised to help me to analyse my video recorded lessons in order to understand the types and functions of gestures that my participating teachers used, in an attempt to facilitate meaning-making in their learners.

Taylor (2014) researched how gestures, postures and words give meaning to students. In his study, Taylor provides "examples demonstrating how embodied modes such as gesture, posture, facial expression, gaze and haptics work in conjunction with speech in children's collaborative construction of knowledge" (p. 401). He emphasised that "children's meaning-making is achieved through the appropriate use of all available semiotic resources" (Taylor, 2014, p, 401). According to van Leeuwen (2004), semiotics resource is the use of signs and symbols to make mathematical representations. In this study, gestures are semiotic resources that participants use to provide meaning making.

Furthermore, Edwards (2009) analysed how mathematics schoolteachers use gestures to convey meaning on the topic of fractions. He further stressed that it is important for the learners to make sense of gestures that the mathematics teachers produce, in order to grasp their mathematical talk.

2.10 THEORETICAL UNDERPINNING

Often researchers in mathematics education use theories to locate their studies (Sriraman & English, 2010). This study is underpinned by enactivism which is fundamentally a theory about autopoietic systems and the biology of cognition (Maturana & Varela, 1987). In the following subsections, the concept of 'enactivism" is put into context of my study.

2.10.1 Enactivism – theoretical underpinning

According to Proulx (2013) enactivism "is an encompassing term given to a theory of cognition that views human knowledge and meaning-making as processes understood and theorized from a biological and evolutionary standpoint" (p. 5). Enactivism claims that human cognition does not occur in minds or brains only, but it "brings together action, knowledge and identity so that there is a conflation of doing, knowing, and being" (Davis, Sumara & Kieren, 1996, p. 152). The biological structure (body, brain and heart) of an individual makes sense of the world in a holistic manner by a process of co-emerging all the sub-structures listed above (Maturana & Varela, 1992). In the context of my study, the teachers' acts of gesturing are an integral component of the body, heart and mind, and of course, the environment.

In my analysis of a teacher's gestures, I was mindful of interpreting the gesture in the context of all these elements, i.e. body, heart and mind, and of course, the environment. A gesture is therefore a manifestation of the mind, body and heart in the context of the environment in which it takes place. Similarly, a learner's interpretation and meaning-making of a teacher's gesture is by being involved in observing the teacher's actions (gestures) in relation to her body, mind, heart and the environment. Enactivism recognizes the biological structure of an organism and the environment (Maturana & Varela, 1992) in relation to meaning-making. In my study, the environment is the mathematics classrooms while teachers and learners are the organisms interacting with each other through gestures in order to communicate mathematical ideas.

In my case, the focus is on establishing how gestures are used to aid the teachers to express mathematical ideas to learners. Simultaneously, the leaners are observing the teacher's gestures, and these gestures are aiding learners to make mathematical meaning. The teacher's entire structure of "the body is involved in thinking and speaking about the mathematical ideas" (Alibali & Nathan, 2012, p. 247) by gesturing in the classroom.

2.10.2 Embodied cognition

Embodied cognition is a branch of the enactivism theory (Morris, Wobbrock & Wilson, 2010), and numerous scholars who work with embodied cognition make different claims. For instance, Glenberg, Witt and Metcalfe (2013) state that the "fundamental tenet of embodied cognition is that […] thinking is not something that is divorced from the body; instead thinking is an activity strongly influenced by the body and the brain interacting with the environment" (p. 573). In other words, the way people think and represent knowledge is influenced and constrained by the human body and the human perceptual system (Wilson, 2002). In addition, Dackermann, Fischer, Nuerk, Cress and Moeller (2017) observe that in the "links between motor activities and cognition, there is accumulating evidence linking the processing of numbers to motor activities" (p. 545). For example, "one of the most intuitive embodied interactions with numbers is finger use, for instance, in counting"(Dackermann et al., 2017 p.546). Similarly, Varela, Thompson and Rosch (1991) state that "the mind is not seen as a complex system of cognitive cogs and levers, but rather as a unified whole, an organism, whose cognitive feats can be described in terms of the non-linear dynamics of dynamic systems theory" (p. 121).
In cognitive neuroscience, "theories of embodied cognition often seem to imply that cognition is embodied, because it recruits neural resources comparable to those used in perception and action" (Van Elk, Slors & Bekkering, 2010, p. 1). Van Elk et al. (2010) also discovered that our brain responds to certain classes of stimuli in cognitive neuroscience. Thus, "the structuring and restructuring of sensorimotor links in the recursive interaction of an organism with its environment, by means of which the organism adapts to it, implies or specifies knowledge of the world" (ibid. p.1). In the next section, I link mathematics teachers' gestures and embodied cognition.

2.10.3 Embodied actions and gestures

Most of the time when the human body is involved in thinking and speaking, people produce actions and those actions are called gestures (Hostetter & Alibali, 2008). This study argues that gesturing is integral to embodied cognition. Several authors have articulated that gestures are embodied cognition. For instance, Hostetter and Alibali (2008) argue that "gestures emerge from perceptual and motor simulations that underlie embodied language and mental imagery" (p. 495). Leman (2012) added that the body acts as a mediator between our environment and the experience and that the body could "build up a repertoire of gestures and gesture/action consequences" (p. 5). He called such a repertoire a gesture/action-oriented ontology.

2.10.4 Co-emergence

Co-emergence is "a situation whereby a change of both, a living system and its surrounding environment depends on the interaction between the system and its environment" (Li, Clark & Winchester, 2010, p. 407). Moreover, Dongwi and Schäfer (2019) conclude "when the system and the environment interact, they become structurally joined" (p. 193). Dongwi and Schäfer's (2019) conclusion agrees with Schäfer's (2016) suggestion that with co-emergence, teaching and learning cannot not be separated, because they are mutually dependent on each other as they develop and grow together (p. 318). In this study I adopt Schäfer's (2016) suggestion as fundamental to my view of teaching and learning when observing teacher's gestures in their mathematics classrooms.

2.10.5 Co-emergent process and gestures

Co-emergence is viewed by many authors as a process whereby teaching and learning takes place together in a situated environment (Kim, Roth & Thom, 2011). In addition, Varela et al. (1991) "see a living organism (person, animal, or plant) and their environment as needing to be considered together, that one cannot separate knowing from doing and from the body, and that knowing is doing which in the end is inseparable from self-identity or being" (p. 121). In this study, the use of gestures, the teacher and the teaching process are therefore viewed as intertwined and co-emergent.

2.11 CONCLUSION

In this chapter, the literature relevant to the study was reviewed. This helped to develop the analytical framework in Tables 3.5 and 3.6 which I discuss in the next chapter. The study of gestures in education constitutes an open field and raises many important questions. Research projects on gestures have the potential to shed light on knowing and learning in everyday school settings. The next chapter discusses the research design and methodology for this study.

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

The aim of this study was to analyse selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making. To achieve this goal, I used different research techniques. In the previous chapter, the relevant literature and theoretical framework for this study was discussed. The purpose of this chapter is to outline the research methodology employed in the study, to address the research objective outlined in Chapter One. This chapter discusses the research orientation, research methodology, data analysis, validity and reliability.

3.2 RESEARCH ORIENTATION

This study was situated in an interpretive paradigm. Qualitative methods of analysing data were employed. Researchers use different approaches to look at the world, make expectations of the world and to understand the world (Hammersley, 2013). These approaches are named as paradigms. The term 'paradigm' originated from the Greek word '*paradeigma*' which means 'pattern' and "was first used by Thomas Kuhn in1962 to denote a conceptual framework shared by a community of scientists, and which provided them with a convenient model for examining problems and finding solutions" (Thomas, 2010, p. 29). Olsen (2004) describes a paradigm as "a pattern, structure and framework or system of scientific and academic ideas, values and assumptions" (p. 16). According to Lather (2004), paradigms are classified into four categories: deconstruction (post- structuralist), understanding (interpretive approaches), prediction (positivism) and emancipatory (critical theoretical approaches). This study adopted the second paradigm, i.e. interpretive approaches, which is discussed further in the next paragraph.

This study, situated in an interpretive research paradigm, aimed at capturing and interrogating the gesturing practices of the selected teachers and their understanding of how they teach, using these gestures. Walsham (2006) highlighted that interpretive research involves capturing social reality as viewed and interpreted by the individual participants according to their own ideological positions. In order to maintain the focus on the phenomenon under investigation, researchers in this paradigm, view the world through the perceptions and experiences of the participants (Thanh, Thi & Thanh, 2015). In this study, I gained a deeper understanding of not only how selected grade 8 mathematics teachers use gestures as visualisation tools to support mathematical meaning, but also how they themselves view their use of gestures in the classroom.

3.3 RESEARCH METHOD

3.3.1 Qualitative case study

A research method, also referred to as a strategy of enquiry, refers to the way in which data are collected and analysed, as well as the type of generalizations and representations derived from the data (Myers & Avison, 2002). A research method can be classified as qualitative or quantitative (Cohen, Lawrence & Keith, 2018) with the difference being in the types of data they generate: quantitative method generate numerical data while the qualitative method data are descriptive, and mainly about phenomenon which cannot be measured, but rather observed. This research is a qualitative case study. According to Hamilton and Corbett-Whittier (2012) a case study "[i]s a way of framing a particular bounded unit, providing guiding principles for the research design, process, and communication" (p. 12). A case study is an appropriate methodology to use where the researcher wishes to understand in some depth, the participants' views and experiences of the problem under investigation. This case study involved interactions with three selected Grade 8 mathematics teachers. The unit of analysis of this research is the teachers' practices focusing on the use of gestures as visualisation tools to support mathematical meaning. The data collection methods for the study, as discussed in detail in Section 3.4, include observations by video-recording five lessons per teacher, and one-on-one stimulus recall interview with each of the three teachers.

3.3.2 Research goal and significance of the study

This study explored how different types of gestures are used by mathematics teachers to develop mathematical ideas in their teaching. This study contributes towards an improved understanding of the link between gestures, meaning-making in mathematics and visualisation. It has also created awareness among mathematics teachers, researchers, policy makers and curriculum designers about the roles and impact of gestures in mathematics education. To achieve this goal, the research was guided by the following objective: to analyse selected Grade 8 mathematics teachers' gestures as visualisation tools to support mathematical meaning-making.

3.3.3 Research question

This study sought to answer the following question:
How do selected Grade 8 mathematics teachers' use gestures as visualisation tools to support mathematical meaning-making?

3.4 RESEARCH DESIGN

Research design is simply the master plan for the research process. Thomas (2010) states that the research design is important for a study because it indicates all the important components of the research process (participants, samples, data collection methods, programs, data analysis techniques, etc.) which help to answer the research questions. The following five phases (summarised in Table 3.1) were adopted for this study:
Phase one: identification of site and selection of participants.
The study was conducted at two schools (a secondary school and a combined school) in the Oshana region of Namibia. Firstly, I secured consent from the folowing: the gatekeeper (shown in Appendix E and Appendix K); the school (shown in Appendix F); selected teachers (Appendix G); and parents of the Grade 8 learners in whose classes I performed the video recordings (Appendix H). The assent form is shown in (Appendix I). The consents were sought after obtaining ethical clearance, which is shown in Appendix D. Using purposeful sampling, three Grade 8 mathematics teachers were selected from the teachers at the two schools. Palinkas et al. (2015) state that purposeful sampling "is widely used in qualitative research for the identification and selection of information-rich cases related to the phenomenon of interest" (p. 1).
Phase two: developing, piloting and refining of data collection and analysis tools.
The pilot study was conducted using the observation schedule and analytical tools in Appendix J, to assist with refining the data collection and analytical tools. For the pilot study, a Grade 7 mathematics teacher at a neighbouring school was considered. After the finalisation of the research tools, enough copies where prepared for the next phases.

Phase three: observation of participants.

Data collection commenced in this phase, whereby the selected participants were observed while teaching. I video recorded five lessons per teacher over a period of three to four weeks, based on the agreed schedules in Appendices A, B and C respectively, showing Analytical Tool 1 Maree (2007) indicates that observations allow researchers to hear, see and begin to experience reality as the participants act (in this case, gesture).

Phase four: interviews.

During the fourth phase, one-on-one stimulus recall interviews, as recommended by Cohen et al. (2007), were conducted between the individual mathematics teachers and the researcher (candidate) in Appendices A, B and C respectively, using Analytical Tool 2. The recorded lessons of the teachers were reviewed together with each of the participants. The discussions, which were audio recorded, were centred on the gestures the teachers used while giving the lessons as a way of understanding why such gestures were used, and importantly, to establish the mathematical meaning-making (if any) that was intended to be achieved. Prior to the interviews, I however, analysed the observation data myself in order to obtain an initial sense of the gestures used by each teacher and be prepared for the joint review of the video recordings.

Phase five: analysis of data

The data collected from the study was analysed and discussed in line with the existing literature, theory and analytical instrument tools in Tables 3.5 and 3.6 to help answer the research question.

Table 3. 1: Summary of the research process

Phases	Instrument	Purpose	Data	Analysis
Phase 1: Selecting of teachers	Consent letters	• Explains in detail the nature of the research project • Get consent from participants (teachers) and then parents of learners	Not applicable	Not applicable

Phase 2: Developing and refining of data collection and analysis tools	Observation and Interview schedules	To ensure readiness for data collection	Not applicable	Not applicable
Phase 3: Collection of data through observations	Observations	Generate data	Types of teachers' gestures, functions and descriptions of actions	Qualitative
Phase 4: Collection of data through interviews	Interviews	Generate data	The roles and functions of gestures as visualisation tools	Qualitative
Phase 5: Analysis of data		Analyse the findings	Video-taped and verbal responses	Qualitative

3.5 RESEARCH TECHNIQUES

In educational research, there are different practices used by researchers to collect data. The research techniques for this study were observations and interviews, which are discussed in detail below.

3.5.1 Observations

Observation is used by researchers to "obtain real-time data about subject's behaviour, interactions, and experiences from the actual environment in which they naturally occur" (Picardi & Masick, 2013, p. 132). The data are first-hand records, because the researcher goes to the site of the study, which in educational settings can be a school, a classroom, or a staffroom (Bertram & Christiansen, 2015). In the case of this study, the setting comprises three mathematics classrooms. Five lessons per teacher were observed i.e. video recorded, over a period of three to four weeks. I sought the assistance of a fellow teacher to do the video recordings for me to observe the teacher and obtain an early sense of what was going on. The video recorder was placed in the back corner of the selected Grade 8 mathematics classrooms in order not to record the faces of any of the learners. The sequence of lessons observed for the three teachers are presented in Tables 3.2, 3.3 and 3.4; for teachers T1, T2 and T3 respectively.

Table 3. 2: Sequence of lessons for teacher T1

Name of the teacher: Mr Kuutondokwa		
Date	**Lesson**	**Topic taught**
13.02.2020	1	Four basic operation
25.02.2020	2	Directed numbers
02.03.2020	3	Properties of number
03.03.2020	4	Power and Base
04.03.2020	5	Four basic operation

Table 3. 3: Sequence of lessons for teacher T2

Name of the teacher: Ms Kambuta		
Date	**Lesson**	**Topic taught**
17.02.2020	1	BODMAS
02.03.2020	2	Division and Multiplication of factor
04.03.2020	3	Directed numbers (integers)
05.03.2020	4	Integers in everyday life
06.03.2020	5	Directed numbers

Table 3. 4: Sequence of lessons for teacher T3

Name of the teacher: Mr Katamba		
Date	**Lesson**	**Topic taught**
15.06.2020	1	Irregular polygons
15.06.2020	2	Angle properties of irregular polygons
16.06.2020	3	Vectors or Transformations
16.06.2020	4	Vectors or Transformations
18.06.2020	5	Vectors

3.5.2 Interviews

As already mentioned, after video recording the five lessons of each teacher, I conducted an initial analysis of the lessons using my Analytical Tool in Table 3.6. I then engaged the participating teachers in one-on-one stimulus recall interviews. Stimulus recall interviews are interviews that reflect what is happening in the lessons, using the video recordings of the lessons as stimulus for discussion and elaborations (Mackey & Gass, 2016). The aim of these interviews was to verify my initial analysis of their use of gestures, and their own understandings of why and how they used gestures to communicate mathematical ideas.

3.5.3 Research site and participants

This study took place at two selected schools in the Onamutai circuit in the Oshana region in Northern Namibia. Figure 3.1 shows the geographical location of Oshana region in Namibia. Tuhafeni Junior Secondary School offers Grade 8-12 while Tutaleni Combined School offers Grade 1-9. All the participants for this study were purposely selected. With purposive sampling, the researcher makes specific choices about which people or groups to include in the sample (Bertram & Christiansen, 2015). The criteria used to select participants were as follows: (1) Grade 8 mathematics teachers, (2) interested in the gesture research, (3) willing to volunteer for the study so that I couldn't observe (video record), and (4) be available for a stimulus recall interview to share their mathematical thoughts on gestures used during their lessons.

Two of the Grade 8 mathematics teachers from Tuhafeni Junior Secondary School (T1 & T3), while the other teacher (T2) from Tutaleni Combined School participated in the study. No sampling technique was employed to select the participants among the Grade 8 teachers, because both schools had only one Grade 8 mathematics teacher. The three teachers offered to volunteer in this study and none of the participants withdrew from the study.

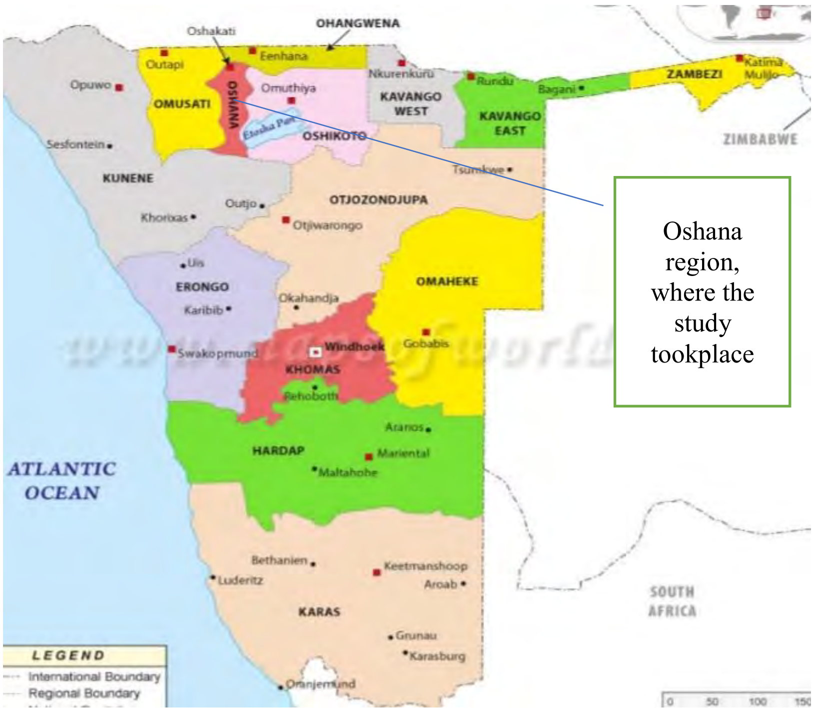

Figure 3. 1: The Namibian political map showing different regions

3. 6 DATA ANALYSIS

The data were analysed qualitatively. Cohen et al. (2018) define qualitative data analysis as a process of organizing, accounting and explaining data in terms of the situation, noting patterns, themes and categories. All the recorded lesson videos and stimulus-recall interviews were analysed using analytical tools (see Tables 3.5 and 3.6). Each lesson was analysed in terms of the five categories of gestures in Table 3.5. Each of these categories were then analysed in terms of the four Rosborough (2010) functions, viz. joint-attention (JA), interpersonal and intrapersonal communication (IC), vocabulary and content development (VCD) and transformation of activities (TA). The stimulus-recall interviews (and their transcripts) were qualitatively studied using the analytical framework in Table 3.5. I have identified strategic gesturing moments in my own analysis of the video recordings. These moments were then visited with each teacher and used as a stimulus to reflect on the gestures used. I asked questions such as: *Why did you use this gesture? What mathematical ideas were you trying to communicate? How did this gesture help you to communicate mathematical ideas?*

Table 3. 5 The Analytical Tool 1 for the observation

Teacher name:		**Grade:**		
Topic taught:		**Date:**		
Five types gestures	**Location in the video recording**	**Description of the gesture and comments.** (McNeill, 1992)	**Functions and actions of the gestures.** (Rosborough, 2010)	**Mathematical meaning**
Pointing Gestures that point or indicate something.	e.g. 8:12-8:18	e.g. pointing at the board at two points seems like he wants learners to make a choice between two options.	e.g. JA and VCD – the teacher is telling learners to substitute in the formulas, numbers at one of the positions on the board.	Mathematical substitution
Metaphor “Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about” (McNeill, 2006, p.23).	g.g. 8:30-8:33	e.g. moving two closed hands away from each other.	e.g. VCD and IC – Illustrate to learners that arithmetic progressions have common differences	Common differences
Iconic Imitate something by physically looking similar.				
Beating “[a]re motions that help emphasize or keep a rhythm in a person’s speech” (McNeil, 2006, p.23).	e.g. 8:35-8:40	e.g. head scratching	e.g. IC and scratching in the head to ask learners to engage in deep thinking when answering the question.	
Overlapping gestures These are two or three gestures that overlap and are not discrete.		(Pointing & beating gestures)		

Table 3. 6 The Analytical Tool 2 for the interviews

Teacher name: **Topic taught:**		**Grade:** **Date:**
Location in the video recording	**Types gestures and functions**	**Transcripts of interviews**
8:12-8:18	Pointing – JA and VCD	In this block I will paste the excerpts of the interviews relating to the 8:12-8:18 section of the video recording and add my interpretation of this excerpt in terms of the types and functions of the gestures used.
8:30-8:33	Metaphor – T A	
8:35-8-40	Beat –IC	
Etc.....		

3.7 VALIDITY AND RELIABILITY

In this study, I used triangulation as described by Gay, Mills and Airasian (2012) to validate the data collected. This was done by employing lesson video recordings, audio recordings and stimulus recall interviews. In order to ensure the validity of my research project, I piloted the interview and observation schedules. I also trialled the position of the video recorder with a Grade 7 mathematics teacher at a neighbouring school, to ensure effective and productive filming. Furthermore, I took my findings and conclusions to the participants to confirm whether they agree or disagree with my conclusions.

Bell (1993) indicates, "Validity tells us whether the instrument measures what it is supposed to" (p. 65). Bless and Higson-Smith (2000) explain that the reliability of measurements is the degree to which that instrument produces equivalent results for repeated trials. Although to a certain extent there may be errors, the results of a quality instrument should show a great deal of consistency over a period. To have consistency in the results I selected three Grade 8 mathematics teachers and each teacher was video recorded five times.

There are always threats to validity and reliability, such as when the participants give different answers to the same question at different times, and when the answered are biased (Fink, 2015). For example, the participants may come prepared on what gestures to use in the classroom during observation. Another threat is that there is possibility for participants to withdraw from the study. This was mitigated by explaining the aims and objectives of my project to all participants in order to ensure they understood the research undertaken. The data was interpreted using analytical frameworks and based on underpinning theory, by observing how participants communicated mathematical ideas using parts of their bodies.

3.8 ETHICS

3.8.1 Respect and dignity

Respect "is a word like 'empathy', 'love' and 'compassion', that everyone agrees connotes a positive attribute" (Spagnoletti & Arnold, 2007, p. 707). Spagnoletti and Arnold (2007) add that respect can be viewed differently, depending on the context being discussed and on the cultural background of the people. On the other hand, dignity can be considered both subjectively (considering individual differences and idiosyncrasies), and objectively (as the foundation of human rights) (Gallagher, 2004, p. 587). Gallagher (2004) further states that respect and dignity are linked inseparably link. Thus, in this study, the participants were informed about their participation in the study and they were made aware that they could withdraw anytime. The participants were also informed that their privacy and sensitivity would remain protected, as the information generated from the research would not be disclosed to third parties. This is indicated in Appendix D (Ethics Approval letter), and in Appendix G (consent letter for the participant teachers). Information that is more detailed is in the ethical proposal residing with the Ethics Committee. The identities of participants were also protected to prevent them from being traced in the future. With reference to Appendix D, all the data remain confidential between the researcher and supervisor,

3.8.2 Transparency and honesty

Heise (1985) defined the term transparency "as the deliberate attempt to make all legally releasable information available whether it is positive or negative in its nature in a manner that is accurate, timely, balanced, and unequivocal all with the purpose of enhancing an audience's ability to reason and to hold organisations accountable for their actions, policies, and practices" (p. 214). Honesty is defined as "valuing the need to present information truthfully in all contexts and report the outcomes" (Johnson, Haigh & Yates-bolton, 2007, p. 367). In this study, official written permission to carry out the study was obtained from all stakeholders in the research process, from the Director of Education to the school principals, teachers and parents. The nature of the study and the description of the participants' involvement (duration and the activities to be carried out) are clearly outlined in Appendices E, F, G, H and I respectively.

3.8.3 Accountability and responsibility

Accountability is an important ethical issue in scholarly analysis, managerial, political and in administrative work. Accountability is defined as "the process by which people are able to hold government to account" (Eyben, 2008, p. 8), while responsibility "is a state or fact of being accountable or to blame for something" (Nissenbaum, 1994, p. 77). Thus, in this analysis, the collected data are kept in a safe place on my password-protected computer and in a cloud-based repository. Participants were not exposed to any physical or mental stress, thus they were respected and treated fairly as referred to in Appendix D.

3.8.4 Integrity, academic professionalism, and researcher functionality/positionality

The findings are reported in a complete and honest way, without tampering with the data and their analysis. This research is my work, and in cases where other people's ideas are used, such sources are fully acknowledged. The Junior Secondary Phase suits this study because in my experience, visualisation is emphasised at this phase and many images are expressed as gesture. This research helped mathematics teachers to bring abstract mathematical thoughts into the classroom. During the study, I was aware of my position as a mathematics teacher in the same circuit as my participants, and as a result, I assured them that my position would not disturb their teaching in any way, nor should it compromise their responses, as referred to in Appendix D.

3.9 CONCLUSION

In this chapter, the research methodology and orientation were discussed, whereby I described the qualitative case study as situated in the interpretive paradigm. I also discussed the data collection techniques used to respond to my research question, and how they were piloted. The sampling methods and the approach adopted for data analysis have been described. The issues of data validity as well as ethical considerations were also discussed. In the next chapter, the data generated from observation and interviews are presented, analysed, and discussed.

CHAPTER FOUR

ANALYSIS AND DISCUSSION OF RESULTS

4.1 INTRODUCTION

This chapter presents and discusses the findings of my research project. The data presented in this chapter where gathered from fifteen video-recorded lesson observations and stimulus-recall interviews. The preliminary sections of the chapter include profiles of each teacher, brief overviews of the five lessons of each teacher, and summaries of the interviews of the teachers' transcripts. The key findings are then discussed, focusing on the gestures used by the teachers. The research results have also been intertwined in the discussion section.

4.2 TEACHERS' PROFILES

The study focused on three Junior Secondary mathematics teachers from three separate schools in the Oshana region, located in the northern part of Namibia. The real names of both the teachers and the schools are not revealed in the study, in order to maintain the anonymity and privacy as promised in Chapter Three. Pseudonyms are thus used for each teacher and school. Table 4.1 shows a summary of the teachers' profiles. The first teacher (T1) is Mr Kuutondokwa, a male Head of Department (HoD) of mathematics and science at Tuhafeni Junior Secondary School. The school is located in the Onamutai circuit and accommodates learners in Grades 8 - 12. Mr Kuutondokwa is a highly experienced teacher with 27 years of mathematics teaching experience. The second teacher (T2) is Ms Kambuta, a female mathematics teacher at Tutaleni Combined School. She has 17 years of mathematics teaching experience. Tutaleni Combined School is a Grade 1 - 9 school in the Onamutai circuit. Lastly, the third teacher (T3) is Mr Katamba, a male mathematics teacher at Tuhafeni Secondary School. He is the least experienced teacher, with fifteen years of teaching.

Table 4 1: Profiles of participating teachers

Teachers' pseudonyms	Schools pseudonyms	Sex	Grade (s) taught (mathematics)	Teachers' qualifications	Mathematics teaching experience
Mr Kuutondokwa (T1)	Tuhafeni JSC	Male	Grade 8 – 9	Grade 12, BETD & ACE	27 years teaching experience
Ms Kambuta (T2)	Tutaleni CS	Female	Grade 5 – 9	Grade 12, BETD, ACE & BEd Hons	17 years teaching experience
Mr Katamba (T3)	Tuhafeni SS	Male	Grade 8 – 12	Grade 12, BETD and ACE	15 years of Teaching experience

Key:

B Ed Hons Bachelor of Education Honours

BETD Basic Education Teacher Diploma

ACE Advanced Certificate in Education

4.3 LESSON OBSERVATIONS OF MR KUUTONDOKWA (T1)

The analytical tools used to analyse data for Mr Kuutondokwa (T1) lessons (1-5) are presented in Appendix A. The lesson descriptions below make use of the acronyms alluding to Rosborough's (2010) functions of gestures, i.e. IC – inter- or intrapersonal; JA –attention; and VCD – vocabulary content development; TA – transformation of activities.

4.3.1 Lesson 1 (Four basic operations)

This is the first lesson I observed of Mr. Kuutondokwa teaching Grade 8. He introduced the lesson by asking learners to identify prime numbers between 1 and 100. The first mathematical gesture observed in this lesson was a beating gesture that occurred at 11:20 – 11:21 whereby Mr. Kuutondokwa rhythmically tapped a pointing finger at the learners and asked the learners not to change the rules. This gesture was identified as an IC function of gesture of (Rosborough, 2010). The rule referred to here was 'BODMAS' (Bracket Of Division Multiplication Addition Subtraction). Again, at 11:22 – 11:23, he moved his open hand downward and asked the learners to use division. According to McNeill (1992), gestures and speech must work together to help put the message clearly across to the targeted audience. In this analysis, the targeted audience were the learners that had to use division. This is an iconic gesture and its functions are VCD and TA. The mathematical meaning of that gesture is 'dividing'. In the same lesson at 11:23 - 11:24, Mr. Kuutondokwa moved his left arm from far away towards himself as if he was collecting items together and said "*addition*". This is a metaphoric gesture, and its roles are IC, VCD and TA. In this instance, it means 'addition'. At l1:24 – 11:25 the teacher closed his hands and used his pointing finger to indicate subtraction, and this gesture is illustrated in Figure 4.1. This is a combination of iconic and metaphoric gestures, i.e. an overlapping gesture and its Rosborough (2010) functions are IC, VCD and TA. At 11:32 – 11:33, Mr. Kuutondokwa pointed at 'of' in the mathematics problem '4 + 3 of 8' and moved down as if to create side brackets. Mathematically, he meant multiplication. This gesture is once again a combination of pointing and metaphoric gestures and it facilitated JA, VCD, IC and TA.

In order to avoid misinterpreting the gestures produced by Mr. Kuutondokwa during the lesson, I decided to interview him about his mathematical actions in the classroom to find out what he was trying to communicate mathematically. The sub-section below includes the audio transcripts of Mr Kuutondokwa and the interviewer.

Figure 4. 1: The teacher closes his hands and uses his pointing finger to indicate subtraction

4.3.1.1 Stimulus recall interview for T1, Lesson 1

This section contains the transcript of the interview between Mr Kuutondokwa and the interviewer for Lesson 1.

> DH: *What is the meaning of the gestures you made between 11:20 - 11:21?*
> Mr Kuutondokwa: *It sounds like a warning because it is very serious; when you change then things will not work properly.*
> DH: *When you change the order of mathematics?*
> Mr Kuutondokwa: *Yeah*
> DH: *Is it like a warning?*
> Mr Kuutondokwa: *Yes, a warning for them not to change the order of BODMAS*
>
> **At 11:22 -11:23:**
> DH: *Can you see your right arm and you are mentioning division, what were you referring to?*
> Mr Kuutondokwa: (asked for a replay) *I was referring to division.*
>
> **At 11:23 - 11:24:**
> DH: *Your right arm moved like this as if you are collecting items, what were you referring to?*
>
> Mr Kuutondokwa: *Yah I bring them from that side to one side and add them together. This means addition.*
>
> **At 11:24 - 11:25 (**looking at the video)
> DH: *Here you moved your pointing finger from the rest of the hand. What were you referring to?*
> Mr Kuutondokwa: *This refers to subtraction*
>
> **At 11:32 - 11:33** (looking at the video):
> DH: *You see what you did at 'of', you created brackets, what was that?*
> Mr Kuutondokwa: (sked for a video replay) *It implied that there is multiplication at 'off'.*

4.3.2 Lesson 2 (Directed numbers)

On the 25 February 2020, I observed Mr Kuutondokwa's second lesson. At 10:07 - 10:08, Mr Kuutondokwa pointed at a number line in the classroom and moved his arm pointing from side to side within the number line. He asked his learners where some numbers belonged on the number line, by randomly mentioning those numbers. He used a combination of pointing and beating gestures, i.e. an overlapping gesture. This gesture helps with IC and JA. Mathematically, he was referring to the 'number line'. In addition, at 10:13 - 10:14, he moved his right arm upwards to illustrate 'ascending order'. Figure 4.2 shows that metaphoric gesture which helped with IC, JA, VCD and TA. One other gesture occurred at 10:14 - 10:15 when he moved his right arm from a high point downwards, to mean ‘descending order’. This is also a metaphoric gesture and it facilitated IC, JA, VCD and TA. In addition, at 10:15 - 10:16 Mr Kuutondukwa opened his right hand, moved it up and down as if he was balancing something in that hand, and said: “*Equal numbers have the same value*”. That metaphoric gesture provided VCD and TA functions. Mathematically, he was referring to ‘equal numbers’. In the same lesson, a beating gesture was observed at 10:24 - 10:25 when he moved his right arm away from himself and said *“… bigger number”*. The functions of this gesture are VCD, IC and TA. Mathematically, he meant ‘greater than’. In this lesson, Mr Kuutondokwa was using both gestures and verbal communication to resolve multiple meanings. This elicited students’ justification of their thinking and, most importantly, advanced the mathematics lesson (Castellon & Enyedy, 2006).

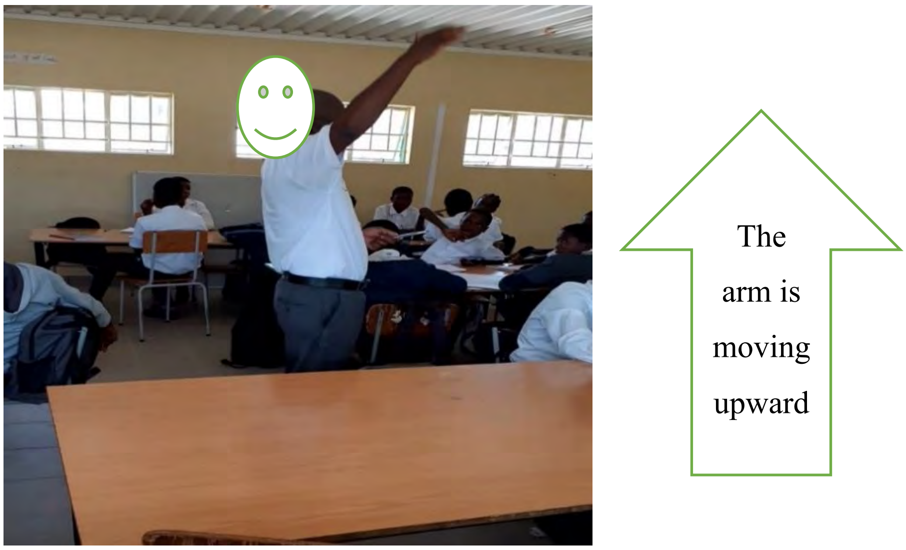

Figure 4. 2: The ascending order gesture

4.3.2.1 Stimulus recall interview for T1, Lesson 2

This section contains the transcript of the interview between Mr Kuutondokwa and me for Lesson 2.

At 10:07 - 10:08 (replaying the video):
DH: *Look at the video you pointed at the number line, what mathematical meaning are you referring to?* Mr Kuutondokwa: *The number line displays positive and negative number; they need to know that from the origin to the right is positive numbers and from the origin to the left is negative numbers".*

At 10:13 - 10:14:
DH: *You see your action, what were you referring to?*
Mr Kuutondokwa: *I told learners that when you are moving from small to big you are rising.*

At 10:14 - 10:15:
DH: *Now there is action again, for descending order.*
Mr Kuutondokwa: *Descending is from big to small, i.e. you are dropping.*

At 10:15- 10:16 (replaying the video):
DH: *You see you opened your hand like this, what were you referring to?*
Mr Kuutondokwa: *There I meant that the numbers are equal in magnitude.*

At 10:24 - 10:25:
DH: *Here, you moved your arm and say big number. What does this mean?*
Mr. Kuutondokwa: *That one is wrong there is no such a thing.*

4.3.3 Lesson 3 (Properties of number)

This is the third lesson of Mr Kuutondokwa that I observed. Only three mathematical gestures were observed in this lesson, and the first two happened at 08:01 - 08:02 when he moved his hands together and said "*addition*". Figure 4.3 below illustrates the first gesture of this lesson. Moreover, during the second gesture, Mr Kuutondokwa pretended to be taking something from one hand and said "*minus*". These two gestures are metaphoric, and they help with VCD. The VCD provides learners with clues on how to tackle a mathematical problem. According to Alabali and Nathan (2012) "… teacher uses gestures to indicate or request objects before they produce words for these referents" (p. 248). Mathematically, he referred to 'addition' and 'subtraction' respectively. The third gesture happened at 08:02 - 08:03 when he turned his hand upside down and said "*inverse/opposite*" of a number. This metaphoric gesture provides a bridge between image and speech and a nexus bringing together actions, imagery, memory and mathematical problem solving (Edward, 2005). Mr Kuutondokwa provided the bridge with the actions of ‘inverse’, so that this could create mathematical memory in the learners’ minds. The Rodborough’s functions for that gesture are VCD and TA.

Figure 4. 3: Gesture demonstrating addition

4.3.3.1 Stimulus recall interview for T1, Lesson 3

This section contains the transcript of the interview between Mr Kuutondokwa and me for Lesson 3.

> **At 08:01 - 08:02** (replaying the video):
> DH: *You see there you have moved the two hands together before adding two numbers do you have any comment on that?*
> Mr Kuutondokwa: *No, it’s just a matter of putting together, i.e. addition.*
> (Looking at the video again)
> DH: *You see what you did before you minus.*
> Mr Kuutondokwa: *That simply meant take away, i.e. subtraction.*

> **At 08:02 - 08:03:**
> DH: *What were you referring to here?*
> Mr Kuutondokwa: *Inverse*

4.3.4 Lesson 4 (Power and base)

The first mathematical gesture in Mr Kuutondokwa’s fourth lesson occurred at 08:06 – 08:07. This was an iconic gesture whose function was VCD. He anchored his fingers on the table to imitate a tree with his fingers representing the roots of a tree, and said, “*Square root or cube* *root*". Mathematically, he meant 'square root' or 'cube root'. Mr Kuutondokwa used gestures of roots to generalise and transfer the idea of 'roots' into mathematical knowledge. Cook et al. (2017) state that the use of gestures helps learners to transfer and generalise their mathematical knowledge into real life problems. At 08:07 - 08:09, two metaphoric gestures were observed. The first gesture happened when Mr Kuutondokwa drew brackets in the air with both hands and said, "*You do multiplication*". This helped with VCD and TA. Here, it referred to 'multiplication'. In the second gesture, he moved his right hand in front of him diagonally downwards and said "*diagonal*". This gesture aided Mr Kuutondokwa to supplement his spoken terms to ensure that the learners understood the concept of 'diagonal' (Flevares and Perry, 2001). The function was VCD, IC and TA. Mathematically, he wanted to illustrate a 'diagonal' line.

At 08:10 – 08:11, an overlapping (beating and iconic) gesture was observed. Mr Kuutondokwa opened his right hand with his fingers meeting at the tips and said, "*We have a symbol"*. This helped with VCD and IC. Mathematically, he meant 'symbols' or 'letters'. The last gesture that occurred during this lesson was a metaphoric gesture (shown in Figure 4.4) that happened at 08:12 - 08:13 when Mr Kuutondokwa opened his right hand with the fingers spread out, moved his hand downwards, and said "*five*". This gesture showed important information about the size of numbers (Chue et al., 2015). Mathematically, he was referring to 'number five'. This gesture helped with VCD and IC.

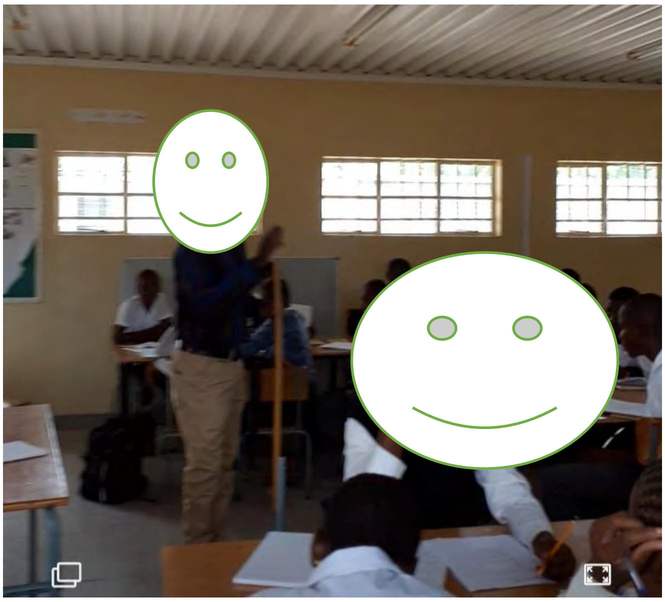

Figure 4. 4: Gesture referring to the quantity 'five'

4.3.4.1 Stimulus recall interview for T1, lesson four

This section contains the transcript of the interview between Mr Kuutondokwa and me for the fourth lesson.

> **At 08:06 - 08:07** (replaying the video):
> DH: *You see you make a root like that, what mathematical meaning were you referring to?*
> Mr Kuutondokwa: *"I was generally referring to the root. When something is growing, a root is where the body is. A root is a number that make up a square; roots are now referring to natural roots."*
>
> **At 08:07 - 08:08:**
> DH: *Can you see the bracket you drew?*
> Mr Kuutondokwa: (asking for a replay) *Just to say what is included inside when you are multiplying.*
>
> **At 08:08 - 08:09**:
> Mr Kuutondokwa: *That means cutting through. I was showing that the line is not straight but a cut through, i.e. a diagonal line.*
>
> **At 08:10 - 08:11** (playing the video):
> DH: *We have a symbol?*
> Mr Kuutondokwa: *Just to indicate that it is significant.*
>
> **At 08:12 - 08:13:**
> DH: *See when you said five?*
> Mr Kuutondokwa: *With that, I was representing number five.*

4.3.5 Lesson 5 (Four basic operations)

This is the last lesson for Mr Kuutondokwa that I observed. The lesson topic was 'four basic operations', and the only mathematical gesture was observed at 08:08 - 08:09. He opened his right hand as if he was indicating that there is nothing in his hand and said "*times zero*". This gesture is illustrated in Figure 4.5 and is classified as a beating gesture whose functions are VCD, IC and TA. Mathematically, he was referring to 'nothing' or 'zero'. This gesture helps the learners to make links between a representation and mathematical thought (in this case 'zero') (Yeo et al., 2017).

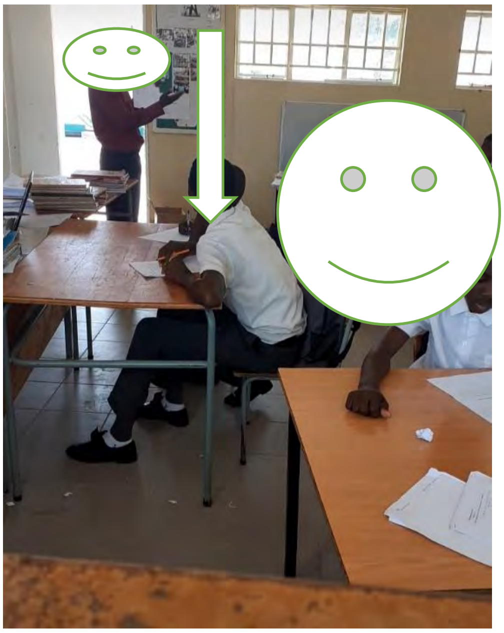

Figure 4. 5: Open right hand moving down

4.3.5.1 Stimulus recall interview for T1, Lesson 5

This section contains the transcript of the interview between Mr Kuutondokwa and me for Lesson 5.

> **At 08:08 - 08:09:**
> DH: *When you say times zero?*
> Mr Kuutondokwa: *They have to leave out because there is nothing.*

4.4 LESSON OBSERVATIONS OF MS KAMBUTA (T2)

The analytical tools used to analyse data for Ms Katamba's (T2) lessons (1- 5) are presented in Appendix B.

4.4.1 Lesson 1 (BODMAS)

The first lesson I observed for Ms Kambuta was on BODMAS and it occurred between 09:20 and 10:00. At 09:27 - 09:28, Ms Kambuta moved her right hand in the air, making 'brackets' on both sides of the number next to the chalkboard, and said "*times*". Figure 4.6 below shows that Ms Kambuta's second gesture occurred at 09:27 - 09:28 – a gesture denoting 'multiplication' in mathematics. This is a metaphoric gesture and it helped with JA and VCD. At 09:33 - 9:34, she rhythmically opened her right hand facing downwards and said "*no calculators*". This is beating gesture and it helped with IC between the teacher and learners.

She was articulating to the learners that the use of calculators was not allowed. Ms Kambuta used her hand as a readily available visual tool to portray a calculator. Other educators also reportedly use readily available visual tools (gestures) (Castellon & Enyedy, 2006) to convey and discuss mathematical algebra concepts. Additionally, Maschietto and Bussi (2008) used gestures to draw and construct pyramids. Furthermore, at 09:42 - 09:43 she made brackets in the air on close to one learner's book and said: "*Use brackets*". This is a metaphoric gesture and its Rosborough (2010) functions are VCD, IC and JA. In this lesson, it was observed that Ms Kambuta first gestured before producing mathematical language, i.e. each mathematical action that she made in this lesson was followed by mathematical terminology.

Figure 4. 6: Brackets created in the air next to the chalkboard

4.4.1.1 Stimulus recall interview for T2, Lesson 1

This section contains the transcript of the interview between Ms Kambuta and me for Lesson 1.

> **At 09:27 - 09:28 (**replaying the video):
> DH: *You see that action, you see you did this, what were you referring to?*
> Ms Kambuta: *I am telling them to put a division sign.*
> DH: *To divide?*
> Ms Kambuta: *Yah you know the opposite of multiplication is division.*
>
> **At 09:39 - 09:40:**
> DH: *What were you trying to communicate to the learners when you did this?*
> Ms Kambuta: *Put down the calculator.*

At 09:48 - 09:49:
DH: *You create brackets there, what are you referring to mathematically?*
Ms Kambuta: *I was referring to brackets; putting brackets indicates which one to answer first according to BODMAS.*

4.4.2 Lesson 2 (Division & multiple factors)

In the second lesson for Ms Kambuta, two mathematical gestures were observed. The first gesture occurred at 10:04 - 10:05 when she opened her arms wide (see Figure 4.7), as if she was hugging, and said "*whole number*". This is a metaphoric gesture and its Rosborough (2010) functions are VCD and IC. Mathematically, she was referring to 'whole numbers'. The second gesture happened when she started nodding her head and asked the learners to find the number of decimal places. This is a beating gesture, and it helps with TA and VCD. Mathematically, it means 'decimal places' The gestures produced by Ms Kambuta in this lesson were not assessed to see whether they made sense to the learners, even though Edwards (2009) stresses that it is important for learners to make sense of the gestures that the mathematics teachers produce, in order to grasp the mathematical concepts they are trying to convey.

Figure 4. 7: The teacher opening her arms as if she is hugging the chalkboard

4.4.2.1 Stimulus recall interview for T2, Lesson 2

This section contains the transcript of the interview between Ms Kambuta and me for Lesson 2.

At 10:04 - 10:05 (replaying the video):
DH: *You see here you moved your arms as if you were hugging, what mathematical meaning are you referring to?*
Ms Kambuta: *This means the whole number.*
DH: *Let us move on to our next gesture. Here you nodded your head and asked the learners how many decimal places. Can you comment on this?*
Ms Kambuta: *I was asking the learners to look at the chalkboard and find the number of decimal places.*

4.4.3 Lesson 3 (Directed numbers)

Ms Kambuta's third lesson, on 4 March 2020, was about directed numbers. Multiple mathematical gestures were observed in this lesson. The first gesture occurred at 10:06 - 10:07 when she made her body look like number line by stretching both her right and left arms to the sides to explain positive and negative numbers. This metaphoric gesture is illustrated in Figure 4.8 and it helped with TA, JA and IC. Mathematically, it means a 'number line'. At 10:07 - 10:08, Ms Kambuta bent both her arms at the elbows by bringing her hands close to her chest and said " … *less than or greater than"*. This gesture, in Figure 4.9 below, shows the teacher in the classroom changing her physical position in order to make mathematical meaning. Maturana and Varela (1992) stated that enactivism recognises the biological structure of an organism and the environment in relation to meaning making. The environment here was the mathematics classroom while Ms Kambuta and her learners were the organisms interacting with each other. The Rosborough (2010) functions of gestures in this iconic gesture are TA, IC, VCD and JA. Thirdly, at 10:09 - 10:10, she moved her left hand far away from her head and related numbers as she demonstrated the number line. This enabled TA, IC and VCD. The overlapping gesture (pointing and metaphoric) that occurred at 10:09 - 10:10, meant that the number line is infinite, but due to a lack of space, it ended there. Moreover, during the lesson at 10:24 - 10:26, Ms Kambuta moved her right hand up and down and said "*Above and below sea level*" respectively. In addition, she further indicated that below sea level was from her hip downwards while above sea level was from her hip upwards. That metaphoric gesture facilitated TA, IC and VCD. Mathematically, it means 'positive' and 'negative numbers' on a number line.

At 10:27 – 10:29, Ms Kambuta pointed at a drawing of the sea on the chalkboard and said, "*The drawing is like a number line and where the water end at the drawing it is zero and when going down the water level is negative and when going up the water level is positive number"*.

The gesture that occurred at 10:27 – 10:29 was an overlapping gesture (the combination of pointing and beating gestures) with its mathematical meaning being the 'number line'. The functions of that gesture are VCD, TA and IC. At 10:32–– 10:34, an iconic gesture, whose Rosborough (2010) functions are TA, VCD AND IC, was observed when she moved her right hand away from her left hand and said "*take out*". She was referring to 'minus' in mathematics. Moreover, at 10:36-10:38, pointing and beating gestures were observed together, i.e. an overlapping gesture, when she pointed and beat at a number line while at the same time moving to the right side gradually to demonstrate addition. The functions for this gesture are TA & IC.

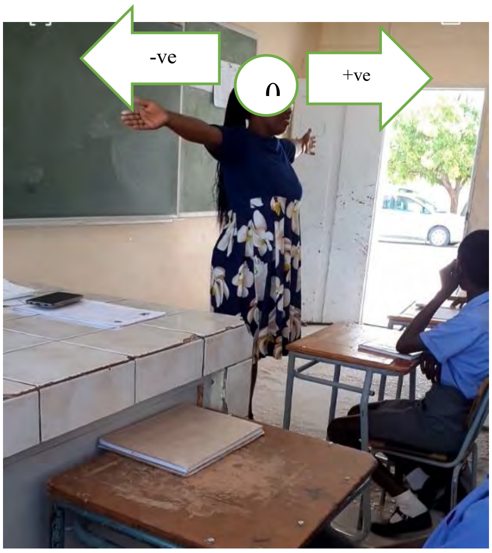

Figure 4. 8: Number line

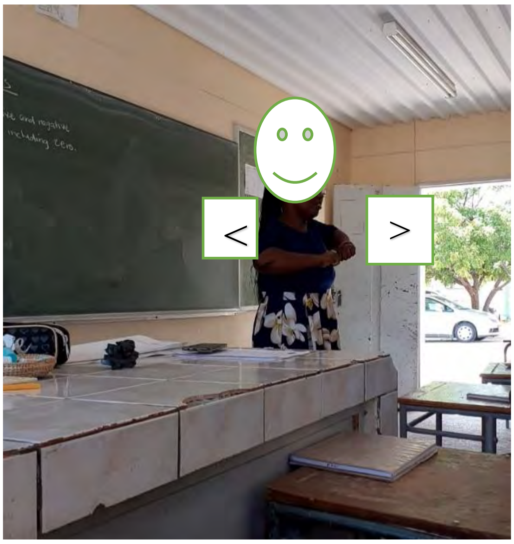

Figure 4. 9: Greater than or less than

4.4.3.1 Stimulus recall interview for T2, Lesson 3

This section contains the transcript of the interview between Ms Kambuta and me for Lesson 3.

At 10:04 – 10:07:

DH (replaying the video): *With that action, what mathematical meaning were you referring to?*

Ms Kambuta: *When doing my hand like this, I am representing my head to be zero and as number goes to the left, the numbers become smaller and as number goes to the right, the numbers become bigger. It means the numbers are increasing as you move to the right side and they are decreasing as you move to the left side, from my body.*

At 10:07 - 10:08:

DH: *Look, what mathematical meaning are you referring to?*

Ms Kambuta: *Greater than and less than.*

At 10:09 - 10:10:
DH: *You see what you did?*
Ms Kambuta: *The arrows, meaning the number line continue what limit it is just the space.*

At 10:24 - 10:26:
DH: Here, here
Ms Kambuta: *For the learners to understand that the hip represents the zero. Below it, numbers go down and above, the number go up. For the sea level diagram, when the water rise the numbers are increasing and when the level is dropping, it means the numbers are decreasing.*

At 10:27 - 10:29:
DH: *Here you have pointed at the level of water, what were you referring to?*
Ms Kambuta: *The level of water show the learners that when you are going deep the water you are going below the sea level (i.e. negative numbers) and when you are going above the sea level it represents positive numbers.*

At 10:32 - 10:34:
DH: *Look*
Ms Kambuta: *I meant subtraction*

At 10:36 - 10:38:
DH: *Moving right arm systematically, what you were doing?*
Ms Kambuta: *It is like when you have negative numbers it is difficult to understand negative plus positive number but when you have it on a number line. The learners understand that if you are at negative two on the number line, three steps to right will end at negative five.*

4.4.4 Lesson 4 (Integers in everyday life)

Three gestures were observed in Ms Kambuta's fourth lesson of. A metaphoric gesture occurred at 8:05 - 8:06 when she moved her left arm vertically with her finger pointing and her right arm moving horizontally, also with an index finger pointing, and said '*positive'* and '*negative'* respectively. Mathematically, she was referring to a 'number line'. This gesture helped with JA and VCD. The second gesture at 08:15 – 08:16 was a pointing gesture where she pointed down and said *"... below water"*, and this is illustrated in Figure 4.10 below. This gesture integrates the JA, IC and VCD Rosborough functions (2010). Mathematically, it means 'negative numbers on a number line'. A beating gesture took place at 08:15 - 08:16 when Ms

Kambuta raised her right arm and said, "*Rise*" to refer to 'increase' in mathematics. Ms Kambuta's arm was observed first going up before mentioning the term *'rise'*. It was clear that Ms Kambuta's body (right arm) was involved in the thinking and articulating of mathematical ideas (Alibali & Nathan, 2012). The functions for this gesture are JA, IC and VCD.

Figure 4. 10: Pointing down to show below sea level

4.4.4.1 Stimulus recall interview for T2, Lesson 4

This section contains the transcript of the interview between Ms Kambuta and me for Lesson 4.

> **At 08:05 - 08:06:**
>
> DH (replaying the video): *You see*
>
> Ms Kambuta: *It is as if when you are at the right-hand side, you point that direction for the learners to know that the numbers on the right hand side also grow bigger. At the negative side, you must point at the left side. The learners will know that a number line has positive and negative numbers.*
>
> **At 08:15 - 8:16:**
>
> DH: *That one?*
>
> Ms Kambuta: *This is above than you go deep, to show the learners that the water level was up then it decreases.*

4.4.5 Lesson 5 (Directed numbers)

This is the last lesson I observed of Ms Kambuta, and numerous mathematical gestures were observed. For example, at 10:03 – 10:04, two metaphoric gestures occurred. The first gesture was when she moved her downward-pointing hand and said "*Below sea level*". This indicates 'negative numbers' on a 'vertical number line' and this gesture helps with VCD. Similarly, she moved her right hand up from her hip and said "*above sea level*" to explain the positive numbers. At 10:04 - 10:05, a beating gesture occurred as she moved both hands from the chest forward repetitively and said, "*increase/rise*". Mathematically, she was referring to 'increase'. A metaphoric gesture was observed at 10:08 - 10:09 when she opened her hand facing downwards and said "*no calculator"*. This helped with IC between the teacher and learners and it helped the learners with VCD. At 10:11 - 10:12, an iconic gesture was observed when she made the '+' sign with her index finger and small finger crossing each other, as seen in Figure 4.12. This gesture helps with VCD, IC and TA. She referred to 'addition' in mathematics. Ms Kambuta also made parallel lines by pointing her fingers at 10:21 - 10:23 and asked the learners to draw their own ("*private"*) number lines. Ms Kambuta clearly understood the concept of parallel lines and could thus gesture appropriately in the classroom. Verela, Thompson and Rosch (1991) indicated that knowing, doing and body actions could not be separate. I classified that gesture as iconic that enables VCD. Mathematically, Ms Kambuta was referring to 'number lines'.

Figure 4. 11: Ms Kambuta making a 'plus' sign with her fingers

4.4.5.1 Stimulus recall interview for T2, Lesson 5

This section contains the transcript of the interview between Ms Kambuta and me for Lesson 5.

At 10:03-10:04:
DH: *You are moving your hands while saying below and above. There are two gestures there. What mathematical meaning were you referring to?*
Ms Kambuta: *It means a child must know that with a negative number, you go down, meaning a child should add when it is below or maybe if it was above, the child should subtract drawback from below.*

At 10:04-10:05:
DH: *This one*
Ms Kambuta: *Increase going up decrease going down*

At 10:08-10:09:
DH: *Without a calculator, you see the action you did.*
Ms Kambuta: *Meaning the learners must not use any calculator.*

At 10:11-10:12:
DH: *You make this "+" sign with your fingers, did you mean addition?*

Ms Kambuta: *No, I was just using my hand, I did mean plus, I wanted them to concentrate.*

At 10:21-10:23:

DH: *You said private number line*

Ms Kambuta: *For the learners to draw their own number line.*

4.5 LESSON OBSERVATION OF MR KATAMBA (T3)

The analytical tools used to analyse data for Mr Katamba (T3) lessons (1- 5) are presented in Appendix C.

4.5.1 Lesson 1 (Irregular polygons)

In Mr Katamba's first lesson, the only gesture that was observed was the metaphoric gesture that occurred at 08:04 - 08:05. Mr Katamba moved his right arm from side to side as if he was making a cross, and asked the learners how many 'x's there were. This metaphoric gesture does not have a physical technique to describe an 'x' (McNeill, 2006). The functions IC and JA were enabled with this gesture. Mathematically it means"x". Figure 4.12 illustrates the gesture in this lesson.

Figure 4. 12: Drawing 'x' in the air

4.5.1.1 Stimulus recall interview for T3, Lesson 1

This section contains the transcript of the interview between Mr Katamba and me for Lesson 1.

At 08:04 - 08:05:

DH (replaying the video): *You moved your right hand that side and asked how many x's are there?*

Mr Katamba: *I was just referring the learner's attention to the picture.*

4.5.2 Lesson 2 (Properties of irregular polygons)

Three gestures were observed in this lesson. The first gesture was a pointing gesture that occurred at 10:04 - 10:05 when Mr Katamba moved his right arm from one angle to another and said, "*The opposite angles are equal"*. The functions for this gesture are JA and IC. Mathematically, it meant 'opposite angles'. The second gesture was the iconic gesture that happened at 10:05 - 10:06 when he drew a circle in the air around a triangle and said, *"All the angles inside are equal to 180 °"*. The Rosborough (2010) functions involved in this gesture are TA, JA and IC. Mathematically, the procedure that he was referring to was 'adding up all the angles in a triangle'. At 10:08 - 10:09, he moved his right arm in a clockwise direction and told the learners that angles must add up in a clockwise direction, as illustrated in Figure 4.13. I classified this gesture as metaphoric and it facilitates VCD, IC, TA and JA. The mathematical meaning of this gesture is 'clockwise movement'. The gestures used in this lesson to help the learners to understand the teacher's instructions (Valenzeno, Alibali & Kitzkya, 2002).

Figure 4. 13: Clockwise rotation

4.5.2.1 Stimulus recall interview for T3, Lesson 2

This section contains the transcript of the interview between Mr Katamba and me for Lesson 2.

At 10:04 -10:05:

DH: *With this gesture, what were you referring to?*

Mr Katamba: *I was referring to opposite angles.*

At 10:05 -10:06

DH: *When you do this, what does it mean?*

Mr Katamba: *I was just referring to the properties of angles in a triangle.*

At 10:09 -10:10

DH: *You see that action?*

Mr Katamba: *Just telling them to follow one direction, either clockwise or anti-clockwise*

4.5.3 Lesson 3 (Vectors & transformations)

The first gesture observed in this lesson was a metaphoric gesture that occurred at 11:01 - 11:02 when Mr Katamba moved his pointing finger sideways and said "*horizontal movement*". This gesture helped with JA, TA, IC and VCD, and its mathematical meaning is 'x axis movement'. The first metaphoric gesture in this lesson is illustrated in Figure 4.14. At 11:03 - 11:04, he moved his hand up and down while holding a mathematical set square. He asked the learners a question: "*How many steps are we moving*?" This is a beating gesture, and it helps with TA, JA and IC. It demonstrates the steps to take, in order to locate the next point of the vectors. Another metaphoric gesture was observed in this lesson at 11:10 - 11:11 when Mr Katamba repeatedly closed and opened one hand while saying, "*Zero is zero*". This means that when the point on a given vector is zero no action should be taken (the position remains the same for that direction). This gesture helps with TA, JA, IC and VCD. This IC was interpersonal communication because it took place between the teacher and learners (Rogers, 1951). Mr Katamba explained to the learners that when the vector is zero, no action is required.

Figure 4. 14: Horizontal movement

4.5.3.1 Stimulus recall interview for T3, Lesson 3

This section contains the transcript of the interview between Mr Katamba and me for Lesson 3.

At 11:01 - 11:02:
DH: *Here?*
Mr Katamba: *Moved across for learners to differentiate between horizontal and vertical movements.*

At 11:03 – 11:04:
DH: *How many steps? You asked while making that gesture.*
Mr Katamba: *I just wanted them to concentrate and be able to see the number of steps moved.*

At 11:10 - 11:11:
DH: *Zero is just zero, what were you referring there?*
Mr Katamba: *Zero, you have nothing, we do not make any step.*

4.5.4 Lesson 4 (Vectors & transformations)

Five gestures were observed in Mr Katamba's fourth lesson. An iconic gesture occurred at 08:05 - 08:06 when he moved his right arm at a distance from his body and asked, "*How many metres*?" Iconic gestures are used to describe features like length (Shein, 2012) as in the first gesture of this lesson. He was referring to the concept of 'length' in mathematics. This provided learners with JA, IC and VCD. Another metaphoric gesture happened at 08:08 - 08:09 when he rhythmically moved his open-handed right arm and said, "*I only give you time*". He was referring to 'time' in mathematics (seen in Figure 4.15). This gesture helps with JA, IC and VCD. The first beating gesture observed in this lesson was at 08:06 - 08:07 when Mr Katamba moved his right arm away from himself with an open hand and widespread fingers and asked: "*What is the direction*?" This gesture indicated the concept of 'direction', and it helped with JA, IC and VCD. The second beating gesture occurred at 08:10 - 08:11 when Mr Katamba moved his right arm and said: "*We are going to move*". He was referring to the moving of vectors and this facilitated JA and TA. Lastly, the pointing gesture occurred at 08:18 - 08:19 when he pointed up and down and said, "*We are going to move up and down*". He was trying to tell learners how to move vectors on the y-axis, which helped with JA, TA IC and VCD.

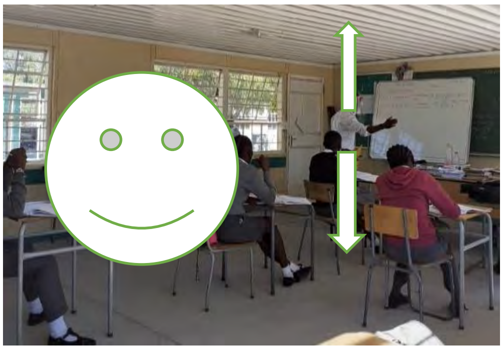

Figure 4. 15: Showing time moving his open hand up and down

4.5.4.1 Stimulus recall interview for T3, Lesson 4

This section contains the transcript of the interview between Mr Katamba and me for lesson 4.

At 08:05 - 08:06:

DH: *You see your arm is moving a distance what were you trying to communicate there?*

Mr Katamba (asked for a replay): *I want to pull their attention to reality of a distance.*

At 08:06 - 08:07:

DH: *Here, can you comment?*

Mr Katamba: *I was demonstrating the concept of direction.*

At 08:08 - 08:09:

DH: *Did you see your arm?*

Mr Katamba: *It was just referring to time.*

At 08:10 - 08:11:

DH: *We are going to move, did you see that action, what were you referring to there?*

Mr Katamba: *Sign of pulling their attention because in vectors we move. In other words, it is a sign of moving or taking steps.*

At 08:18 - 08:19:

DH: *There is this action.*

Mr Katamba: *I was just referring to the movement on the y-axis.*

4.5.5 Lesson 5 (Vector & transformations)

In Mr Katamba's last lesson, the first metaphoric gesture happened at 10:01 - 10:02 when he opened both hands at hip level, as if he was weighing something or carrying a heavy object – as illustrated in Figure 4.16. He then asked the learners, "*Is equal to what?*" This is metaphoric gesture, and it helps with VCD, TA and IC. Mathematically, it means 'equal'. The second gesture of this lesson was a metaphoric gesture at 10:01 -10:02 when Mr Katamba moved his right arm down a bit and said, "*This is one over three*". This gesture helps with VCD, TA and IC, and its mathematical meaning is 'fraction'.

Figure 4. 16: Gesture of equal

4.5.5.1 Stimulus recall interview for T3, Lesson 5

This section contains the transcript of the interview between Mr Katamba and me for lesson 5.

At 10:01 - 10:02:

DH: *Here, what are you referring to?*

Mr Katamba: *I am referring to equal.*

At 10:01 - 10:02:

DH: *You see, what mathematical meaning are you referring to?*

Mr Katamba: *I am referring to fraction[s].*

4.6 DISCUSSION OF THE FINDINGS

This section contains a summary of some interesting findings made during the class observations and amplified during the interviews.

4.6.1 Gestures and speech as an integrative system

Gestures and speech were often observed as an integrative system with all the teachers. For example, Mr Kuutondokwa (T1) in Lesson 1 at 11:23-11:24 moved his left arm from far away towards himself as if he was collecting items, and at the same time said "*addition"*. Similarly, Ms Kambuta in Lesson 5 at 10:03 – 10:04 moved her downward pointing hand and said "*below sea level*". Moreover, Mr Katamba in Lesson 3 at 11:03 - 11:04, moved his hand while holding a mathematical setsquare and asked the learners: "*How many steps are we moving*?" McNeill (1992) asserted that gesture and speech must work together to help put the message clearly across to the target audience. The audience were learners in this study. Furthermore, Castellon and Enyedy (2006) indicated that the use of both gestures and verbal communication resolves multiple meanings, elicits justification for students' thinking and, most importantly, advances the mathematical lesson. While using gestures, the mathematics teachers were justifying their mathematical opinions.

4.6.2 Types of gestures

Multiple types of mathematical gestures were observed in all the lessons of the three teachers (see Table 4.2), with the metaphoric gestures being the most prevalent in all the participants. This suggests that metaphoric gestures can best represent mathematical ideas. As McNeill

(2006) highlighted, metaphoric gestures are used to describe a shape, and to reinforce, affirm, and explain ideas about specific shapes, hence the flourishing of hands, for example, to indicate the size of the mathematical shape. Table 4.2 indicates that in this analysis pointing gestures are used the second most frequently. Interestingly, the overlapping gesture is used least often. In this study, T3 (Mr Katamba) did not use overlapping gestures at all. Table 4.2 also indicates that there is a direct relationship between the years of teaching experience and the frequency of mathematical gestures produced during the classroom. The least experienced teacher used few mathematical gestures in the classroom, compared to the highly experienced teachers.

Table 4 2: Types of gestures observed in the study

Teacher's pseudonym	Pointing gestures	Metaphoric gestures	Iconic gestures	Beating gestures	Overlapping gestures	Combination of overlapping gestures	**Total**
Mr Kuutondokwa (T1)	5	10	1	2	1	I & M	**21**
					1	P & B	
					1	B & I	
Ms Kambuta (T2)	2	8	4	4	1	P & M	**21**
					2	P & B	
Mr Katamba (T3)	3	7	1	3	0		**15**
Total	**10**	**23**	**7**	**9**	**6**		**55**

Key

P – Pointing gestures

M – Metaphoric gestures

I – Iconic gestures

B – Beating gestures

4.6.3 Roles and functions of gestures

Several of Rosborough's (2010) functions of gestures were observed in this study. Table 4.3 summarises Rodborough's (2010) functions observed in the teachers' lessons. The functions included in Table 4.3 are only those specifically used by participants to enhance mathematical communication. This study discovered that when comparing T1 and T2 to T3, Joint or Shared

Attention (JA) was the least important function used to represent mathematics concepts. On the other hand, both teachers often used gestures for Interpersonal and Intrapersonal Communication (IC), Vocabulary and Content Development (VCD) and Transformation Activities (TA).

Table 4 3: Rodborough's (2010) functions observed in this study

Teacher's pseudonym	Joint or shared attention	Interpersonal and Intrapersonal communication	Vocabulary and Content Development	Transformation Activities	**Total**
Mr Kuutondokwa (T1)	2	11	15	11	**39**
Ms Kambuta (T2)	6	11	11	8	**36**
Mr Katamba (T3)	11	12	10	8	**41**
Total	**19**	**34**	**36**	**27**	**116**

4.6.4 Gestures as communication tools

Gesturing is one of the communication tools that can be utilised to transmit information (Valenzen, Alibali & Kitzkya, 2002). In this study, the use of gestures by the teachers to communicate with the learners was observed in all the lessons the researcher attended. One good example of the communication between the teacher and learners using gestures was in Lesson 3 of T2 (Ms Kambuta) when, at 10:24-10:26, she moved her right hand up and down to indicate above and below sea level respectively, while teaching the number line. She was using her hip to represent sea level and then moved her hands up and down to represent positive and negative numbers respectively. Again, Ms Kambuta at 10:27 – 10:29 pointed at a drawing of the sea on the chalkboard and said, "*Where the water ends is like a number line, it is zero, when it goes down the water level is negative, and when it goes up the water level is positive number".*

Furthermore, at 08:12 - 08:13 in Lesson 4, Mr Kuutondokwa opened his right hand with his spread fingers, moved it down, and said *"five"*. Mathematically, he was referring to number 'five'. Amalancei (2014) explains that gestures can only be important communication tools if the receivers (learners in the classroom) understand their meanings. For whatever medium of communication being used it is important that the message passed onto the receiver by the sender should be clearly understood. It would thus be interesting for future studies to interview learners to gauge their understandings about the gestures that were utilised. Such studies could substantiate whether gesturing is an effective communication tool in mathematics classrooms or not.

4.6.5 Gestures as mathematical tools

Edward (2005) indicates that it is important to integrate gestures in the teaching of mathematics. It was observed in this study that some gestures were used as mathematics tools. For example, at 10:36 - 10:38, in Lesson 3 for T2, addition on a number line was demonstrated by moving his hand to the right side. Additionally, in Lesson 5 at 10:01 - 10:02, T3 teacher moved his right arm toward himself, and then moved it down a bit, and said, "*This is one over three*".

Gestures help learners to transfer and generalise their mathematical knowledge into real life problems (Cook et al., 2017). For example, T1 in Lesson 3 demonstrated taking something from his hand and said *"minus"*. The action of this teacher was general and became mathematical when he said *"minus"*.

4.6.6 Gestures as teaching tools

During class observations, it was evident that the three teachers used gestures to facilitate the teaching process. For instance, at 10:15 - 10:16, in Mr Kuutondokwa's (T1) Lesson 2, the teacher opened his right hand, moved it up and down as if balancing something in his hand and said: "*Equal numbers have the same value"*. Mr Kuutondokwa indicated during the interview that he was referring to magnitude. In addition, at 10:04 - 10:05 in Ms Kambuta's (T2) Lesson 2, she opened her arms wide, as if she was hugging, and said: "*Whole number*". Mathematically, she was referring to 'whole numbers'. In the literature, Chue et al. (2015) state that gestures in the classroom show important information about "size, relative position and movement of particles" (p. 1). Chue et al.'s (2015) finding is in concordance with what Mr Kuutondokwa and Ms Kambuta used in their second lessons at 10:15 – 10:16 and 10:04 – 10:05 respectively.

4.6.7 Gestures lighten the cognition load

In this study, the teachers' bodies were observed in trying to help the learners to think. For instance, in Lesson 5 at 10:04 - 10:05, Ms Kambuta moved both her hands away from her chest continuously and said, "… *increase/rise*". Moreover, in Lesson 2 at 10:08 - 10:09, Mr Katamba moved his right arm in a clockwise direction and told the learners that when adding angles together, it should be in a clockwise direction. Again, in Lesson 3 at 11:01- 11:02 Mr Katamba moved his pointing finger sideways and said "… *horizontal movement".* What happened in this study is similar to Alibali and Nathan's (2012) observation that "gestures are taken as evidence that the knowledge itself is embodied and the body is involved in thinking" (p. 247). Similarly, all the participants used their body actions/knowledge to communicate mathematically. As in Alibali and Nathan's (2012) study, I observed that the participants' bodies move first before they communicate mathematical concepts.

4.6.8 Criticism of gestures

In Chapter Two, it was proposed that gestures could be problematic when a mismatch occurs between the gesture and what it is supposed to mean (Church & Golden-Meadow, 1992, Ping et al., 2019). In this study, a gesture mismatch was observed in Lesson 2, at 10:24 - 10:25 when Mr Kuutondokwa (T1) moved his right arm away from himself and said " … *a big number*". This became evident during the stimulus recall interview when the participant indicated that he used the incorrect gesture in this instance. The gesture disrupted the learners, and was thus inappropriate and contained misleading information. Oh (2017) discourages the use of gestures because inappropriate gestures in the classroom can lead to unnecessary laughing and scorning that may leave people feeling embarrassed.

4.6.9 Stereotypes of gestures

In many cultures, people think differently about gesture. For instance, Ntuli (2012) from South Africa states that it is sometimes difficult to understand gestures used by people of different cultures. Thus, culture and socialisation cannot be separated (ibid.). The findings of this study are contrary to Ntuli's findings because the participants of this analysis are from different cultures and different regions in Namibia, viz: Mr Kuutondokwa is a Ndonga (Vambo) from the Oshana region; Ms Kambuta is a Subian from the Zambezi region and Mr Katamba is a Kwambi (Vambo) from the Omusati region. Contrary to the cultural and geographical diversity among the participants, it was not difficult to understand their gestures. Figure 4.17 is a Namibian map showing its fourteen political regions.

From the observations I conducted with my participants from the different cultures identified above, I argue that they all made use of rich gestures. I find it interesting that Kendon's (2004) finding states that some cultures are richer in gestures than others are, even though Kendon's study was also only a case study. In addition, in my study there was a balance between gender and mathematical gestures produced by participants. The only difference between the participants was their years of teaching experience and the mathematical gestures that they used. This was discussed in Section 4.6.2.

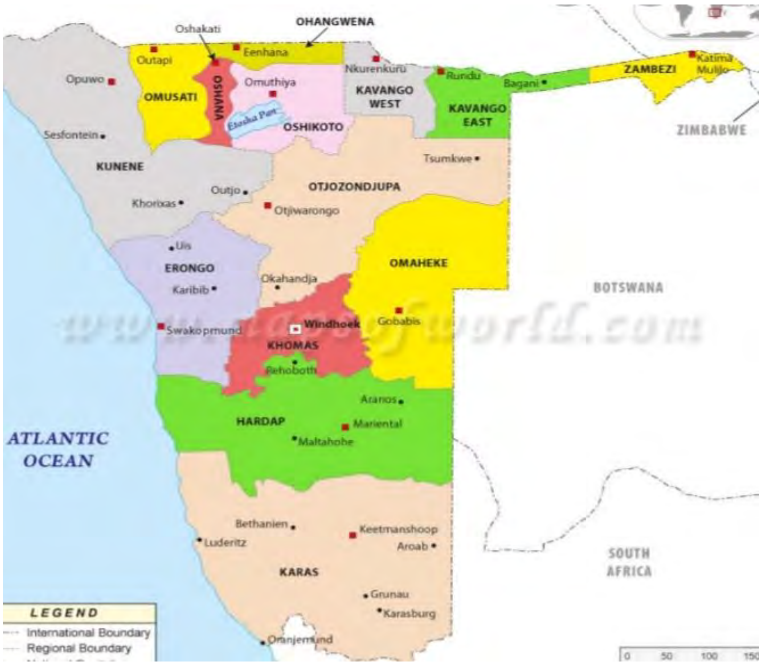

Figure 4. 17: The Namibian map and its regions.

4.6.10 Visualisation tools and gestures

In Chapter Two, it was mentioned that visualisation is viewed "as the ability to represent, transform, generate, communicate, document and reflect on visual information" (Hershkowitz, Arcavi & Bruckheimer, 2001, p. 255). These six visualisation abilities resonate well with my study. All six visualisation abilities were observed on most occasions. For example, T1 in Lesson 1 at 11:22 – 11:23 moved his open hand downwards and asked the learners to use division. The mathematical meaning of that gesture is division. Again, T2 in Lesson 1 at 09:33 – 09:34 rhythmically opened and closed and her downward-facing right hand and said *"... no* *calculators"*. The flow diagram in Figure 4.18, adopted from Hershkowitz et al. (2001), depicts clearly how visualisation was used as a communication tool in the case of Ms Kambuta when she said " … *no calculators"*.

Represent (teacher's gesture (no to calculators)),

↓

Transform (learners try to internalize the teacher's body movement),

↓

Generate (learners start to create the meaning of the teacher actions (teacher does not want us to use calculators)),

↓

Communicate (learners understand the meaning of the teacher action),

↓

Document (learners make connections to the mathematical problem)

↓

Reflect on visual information (learners validate what they have visualised (put down calculators)).

Figure 4. 18: Flow diagram showing the visual process when gestures are used.

.

4.6.11 Meaning making and gestures

In this study, different participants used numerous gestures with the main aim of giving mathematical meaning to the learners. For example, in Lesson 3 at 08:01 - 08:02, Mr Kuutondokwa (T1) moved his hands together and said "*addition*", and in the second gesture he used, he pretended to be taking something from one hand and said "*minus*". The mathematical meaning for Mr Kuutondokwa was 'addition' and 'subtraction'. Moreover, in Lesson 5 at 10:11 - 10:12, Ms Kambuta (T2) made an iconic gesture when she made a "+" sign with her index fingers and smallest fingers crossing each other. She was referring to 'addition' in mathematics. Again, in Lesson 4 at 08:05 - 08:06, Mr Katamba (T3) moved his right arm at a distance from the body and said "*How many meters?"* He was referring to 'length' in mathematics.

Most of the gestures produced by my participants gave mathematical meaning to the learners as discussed in the paragraph above. Edwards (2009) emphasises that it is important for the learners to make sense of gestures that mathematics teachers produce, in order to grasp their mathematical talk and the concepts they are referring to.

4.7 ADDITIONAL FINDINGS

In this study, it was also observed that, at times, different gesturer used same actions to convey the same mathematical meaning. For instance, T1 in Lesson 5 and T3 in Lesson 3 represented "zero" with the same action. This suggests that the rhythmical movement of open hands means "zero or 0".

Another new finding is the gesture used by T2 in Lessons 1 and 5 to instruct learners not to use calculators. This is a clear indication that the "no calculator" gesture with the hand facing downwards while rhythmically beating was well understood by the learners, as they did not have to be reminded verbally to put down their calculators.

This study also discovered a new gesture for 'equal to'. This was gestured by the hands opening as if balancing or weighing something. This action is evident from Mr Kuutondokwa's Lesson 2 at 10:15 - 10:16 as well as from Mr Katamba's Lesson 5 at 10:01 - 10:02. In his lesson, Mr Kuutondukwa opened his right hand, moved it up and down as if he was balancing something in that hand, and said, "*Equal numbers have the same value*". On the other hand, Mr Katamba opened both hands below his hip as if he was weighing something or carrying a heavy object. He then asked the learners "… *is equal to what?"*

A direct relationship between teaching experience and frequency of gesturing was discovered. Thus, T1 and T2 (highly experienced) made more gestures than T3 (least experienced).

It was thus discovered that the more experienced the teachers used more mathematical gestures than the inexperienced teachers did - the findings are illustrated in Figure 4 .19 below.

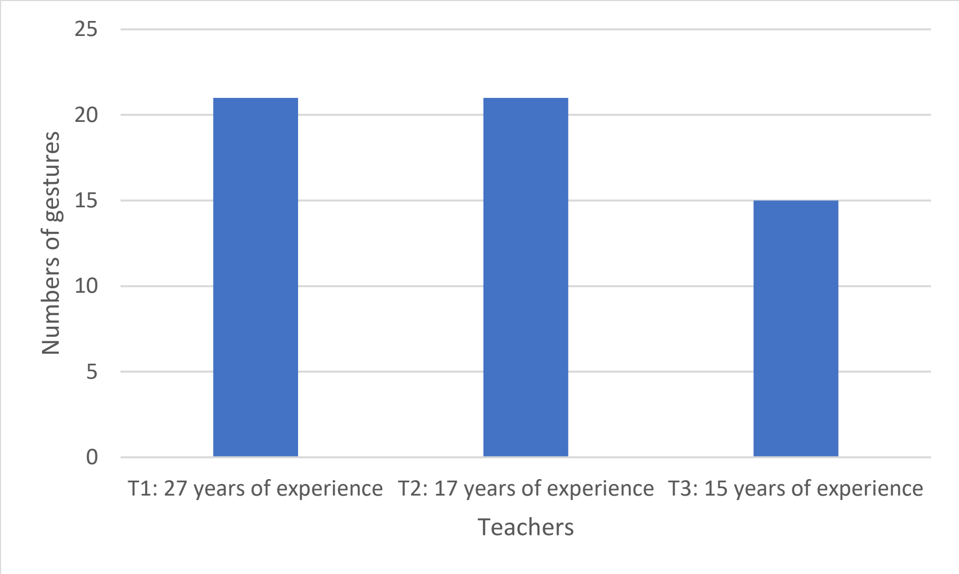

Figure 4. 19: Number of mathematical gestures in comparison to years of teaching experience

4.8 CONCLUSION

This chapter presented and discussed the findings of the research project drawn from fifteen lesson observations and stimulus recall interviews. I first analysed each teacher's lessons and extracted all the gestures that the teachers used in their five lessons. The main findings were then summarised, based on the themes that were used to categorise and discuss the data. The themes came from the analytical tools and some emerged from the interviews.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

In the previous chapter, I presented, analysed and discussed the data gathered in the study. In this chapter, I present a summary of my findings and provide some recommendations. The significance and limitation of the study are also included. Finally, my personal reflection of the research journey is presented.

5.2 OVERVIEW OF THE STUDY

This study sought to answer the following question:
How do selected Grade 8 mathematics teachers' use gestures as visualisation tools to support mathematical meaning-making?
To answer the research question, data was gathered through class observations (video recordings) and stimulus recall interviews. Five lessons per teacher were video recorded. The video recorder was placed at the back corner of each selected Grade 8 mathematics classroom to ensure that the faces of the learners could not be seen in the videos. The recorded videos were initially analysed with the analytical tool in Table 3.5 to identify the gestures used by the teachers, and then the researcher engaged the participants in one-on-one stimulus recall interviews to probe the meanings of the gestures they made during lessons. The interviews verified the initial analysis of their use of gestures, and their own understanding of why and how they used gestures to communicate mathematical ideas.

5.3 SUMMARY OF FINDINGS

The summary of the findings are presented below in relation to the research question.
This study discovered that teachers used gestures and speech as an integrative system to resolve multiple meanings, elicited students' justification of their thinking and, most importantly, advanced their mathematical lessons – which is similar to Castellon and Enyedy's (2006) findings. Moreover, it was discovered that gestures are used as communication tools,

mathematical tools, teaching and learning tools, which all aided with visualisation in the classrooms.

In addition, this study observed that gestures lightened the cognitive load of teachers – thus, the participants' bodies often moved first before they communicated verbally mathematically. A gesture mismatch was also discovered, thus there is a need for strategic use of gestures to avoid misleading the learners. Another interesting outcome of the study was the correlation between frequency of gesturing and level of teaching experience. It became known that the more experienced teachers used more gestures in the classroom than the less experienced teachers. It was, however, discovered that cultural diversity does not influence the use of gestures.

This study discovered three new findings. The first one is the 'no calculator' gesture, which was demonstrated with the pulsing hand facing down. The second one is 'zero or 0' gesture which was demonstrated with a rhythmical movement of open hands. Lastly, the 'equal gesture' shown by moving both hands below the hip as if weighing something or carrying a heavy object.

Among all the types of gestures observed in the classrooms of the participating teachers, the metaphoric gestures were the most used. This is presented in Table 4.2. This suggests that whenever we represent mathematical actions to make meaning we should consider metaphoric gestures. Pointing gestures were also commonly used, but not as much as metaphoric gestures. The least used category of gesture was the overlapping gestures. On the other hand, it became known that Joint Attention (JA), one of Rosborough's (2010) functions, was rarely applied in representing mathematical concepts. However, all the three teachers often used the other Rosborough functions, i.e. interpersonal and intrapersonal communication (IC), vocabulary and content development (VCD) and transformation of activities (TA). Moreover, another finding shown in Table 4.3 was that the Vocabulary and Content Development (VCD) occurred more often. This emerged when the participating teachers made an action followed by a word.

This study discovered a new type of gesture called 'overlapping gesture' that was used by participants to represent mathematical ideas in addition to the McNeill (1992) types, namely: pointing gestures, metaphor gestures, beating gestures and iconic gestures.

5.4 SIGNIFICANCE OF THE STUDY

This study explored how selected mathematics teachers to develop mathematical ideas in their teaching use different types of gestures. This study contributed towards an improved understanding of the link between gestures, meaning-making in mathematics and visualisation. It is also hoped that it will create awareness among mathematics teachers, researchers, policy makers and curriculum designers about the roles and impact of gestures in mathematics education.

5.5 LIMITATION OF THE STUDY

The findings of this case study are not generalisable, neither to all mathematics teachers in Namibia, nor to all mathematics teachers in the Oshana Region, due to the small sample of participants involved. Another limitation is that the participants are all teachers in the Oshana Region, which is also, where I teach. Being observed and interviewed by a colleague in the region could have influenced their pedagogic practice and interview responses. In an attempt to minimise this, I explained clearly the aim of the research to the participants. I also set up my observation equipment in advance and carried out one lesson observation per teacher prior to the actual data collection in order for teachers to begin acclimatising to my presence during the mathematic lessons. Furthermore, the teachers were not observed while teaching the same topics. This might have affected the results, as there is a possibility that some topics have greater potential for the use of gestures than others do.

5.6 RECOMMENDATIONS

Based on the findings of this research, there is a need for mathematics teachers to be trained on how to integrate gestures in their teaching to avoid gesture mismatches that were observed in some instances in this study. I thus recommend that a module on the use of gestures be introduced in the Tertiary Education National Curriculum of teacher training institutions. Alternatively, mathematics teachers should have in-service training to learn more about gestures, and how they can be integrated into their teaching.

Furthermore, I recommend are learners are encouraged to observe mathematics teachers' gestures in the classroom in order to grasp the mathematical concepts that their teachers are trying to convey. Lastly, I recommend that stakeholders such as teachers, policy makers, curriculum designers, senior education officers and heads of departments become involved in understanding gestures and mathematical meanings, and recognise gestures as a legitimate means of communicating and teaching.

5.7 PERSONAL REFLECTIONS

Ever since I completed the BEd Hons at North-west University in 2016, I looked forward to pursuing a degree of Master in Education. In 2019, I began my Postgraduate journey with Rhodes University. I share my research experiences in this journey below.

While it was not an easy journey, even though I had a three-year gap from academic activities, this study however, contributed to my professional growth, especially regarding writing, reading of academic articles, textbooks, and other scholarly sources of information, as well as analysing data. My understanding about gestures and visualisation has improved significantly during the course of my study. Moreover, this study granted me an opportunity to meet fellow researchers from whom I have learnt much about research in general and various research themes in particular.

Briefly, this study has developed me professionally and academically. I am no longer the same David I was before I began this research process, and for that I am pleased. I am grateful to my supervisor for the support, guidance, and patience during this academic journey.

5.8 CONCLUSION

This chapter concludes the study by presenting a summary of findings, discussing the significance of the study and limitations of the study. The recommendations are also outlined. Finally, a reflection of my experiences throughout my entire research journey concludes this chapter and thesis.

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APPENDICES

APPENDIX A: ANALYTICAL TOOLS FOR CLASS OBSERVATIONS FOR MR KUUTONDOKWA (T1)

The analytical tool 1 for the observations (lesson 1)

Teacher name: Mr. Kuutondokwa (T1)		**Grade: 8**		
Topic taught: Four basic operation		**Date: 13. 02-2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	11:23-11:24	*Moving the left arm from far towards him as if he is collecting items to come together and says "addition".*	*IC, VCD & TA - moving the left arm from far until it get s closer to the chest.*	*Addition in mathematics*
Iconic Imitate something by physically looking similar.	11:22-11:23	*Moving open hand downward and ask learners to use division.*	*VCD & TA - moving open hand downward like cutting.*	*Division in mathematics*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	11:20-11:21	*Shaking a pointing finger facing to the learners and ask the learners to use 'BODMAS' and not to change the Rule.*	*IC - shaking the pointing finger in the air.*	*BODMAS*

Overlapping gestures These are two or three gestures that overlap and are not discrete.	11:24-11:25 11:32-11:33	*Iconic & Metaphor -gesture, close hand like making a fist and remove a pointing finger and say 'subtraction'.* *Pointing & Metaphor - pointing at 'of' in a math problem and move down like creating brackets on the side.*	*IC, VCD & TA - a close hand of a fist and remove a pointing finger.* *JA, VCD, IC & TA - pointing at 'of' in the math problem and move down like creating side brackets.*	*Subtraction in mathematics* *Times or Multiplication*

The analytical tool 2 for the interviews (lesson 1)

Teacher name: Mr. Kuutondokwa (T1) **Topic taught: For Basic operation**	**Grade: 8** **Date: 13. 02.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
11:20-11:21	*Beating - IC function*	"*It sounds like a warning because it is very serious; when you change then things will not work properly.* *A warning for them not to change the order of BODMAS"*
11:22-11:23	*Iconic – VCD & TA*	*I was referring to division."*
11:23-11:24	*Metaphor – IC, VCD & TA*	*"Yah I bring them from that side to one side and add them together. This means addition"*
11:24-11:25	*Overlapping (Iconic & metaphor)- IC, VCD & TA*	*"This refers to subtraction"*
11:32-11:33	Overlapping (Pointing & Metaphor) – JA, VCD, IC & TA	*"It implied that there is multiplication at 'off'."*

The analytical tool 1for the observations (lesson 2)

Teacher name: Mr. Kuutondokwa (T1)		**Grade: 8**		
Topic taught: Director Number		**Date: 25. 02. 2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical** **Meaning**

Pointing Gestures that point or indicate something.	10:07- 10:08	*Pointed at a number line in at the chalkboard and moved from side to side within the number line.*	*JA & IC – Moving side by side within the number line.*	*Meaning the number line*
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:13 -10:14 10:14 – 10:15 10:15- 10:16	*Moving his right arm from down level moving up and say ascending.* *Moving right arm from high point to low point.* *Open hands and move them like weighting.*	*VCD, IC, JA & TA – Moving right arm from low point to high point.* *VCD, IC, JA & TA – Moving right arm from high point to low point.* *VCD & TA - Open hands and move them like weighting.*	*Ascending order* *Descending order* *Equal or same number in the value/size*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	10:24 – 10:25	*Moving the right arm from him to a distance, and say big numbers.*	*VCD, IC & TA -Moving the right arm from him to a distance.*	*Greater than*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 2)

Teacher name: Mr. Kuutondokwa (T1) **Topic taught: Director number**		**Grade: 8** **Date: 25.02.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**

10:07 – 10:08	*Pointing gesture and JA & IC functions*	*"The number line displays positive and negative number; they need to know that from the origin to the right is positive numbers and from the origin to the left is negative numbers".*
10:13 -10:14	*Metaphor gesture – VCD, IC, TA & JA*	*"I told learners that when you are moving from small to big you are rising".*
10:14 – 10:15	*Metaphor gesture – VCD, IC, TA & JA*	*"Descending is from big to small, i.e. you are dropping".*
10:15- 10:16	*Metaphor gesture - VCD & TA*	*"There I meant that the numbers are equal in magnitude".*
11:24 -11:25	*Beating gesture – VCD, IC & TA*	*"That one is wrong, there is no such a thing".*

The analytical tool 1for the observations (lesson 3)

Teacher name: Mr. Kuutondokwa (T1)		**Grade:8**		
Topic taught: Properties of numbers		**Date: 02.03.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	08:01 - 08:02 08:01 - 08:02 08:02 - 08:03	*Moving the open hands together and say Addition.* *Act like taking something from the other hand and say 'minuses.* *Turned his hand upside down and said "opposite of a number".*	*VCD & TA – Moving open hands together.* *VCD & TA - Act like taking something from the other hand* *VCD & TA – Turned his hand upside down.*	*Addition* *Subtraction* *Inverse/opposite*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 3)

Teacher name: Mr. Kuutondokwa (T1) **Topic taught: Properties of number**		**Grade: 8** **Date: 02.03.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
08:01-08:02	Metaphor – VCD & TA	*"No, it's just a matter of putting together, i.e. addition".*
08:01-08:02	Metaphor – VCD & TA	*"That simply meant take away, i.e. subtraction".*
08:02-08:03	Metaphor – VCD & TA	*"Inverse"*

The analytical tool 1for the observations (lesson 4)

Teacher name: Mr. Kuutondokwa (T1)		**Grade:8**		
Topic taught: power and base		**Date: 03. 03. 2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	08:07 – 08:08 08:08 – 08:09 08:12 - 08:13	*Draw bracket in the air with both hands and said "you do multiplication".* *Move the straight right arm diagonal from up to him until it goes to the back and say diagonal.* *Opened his right hand with the fingers spread from each other, moved it down and said "five".*	*VCD & TA – Draw brackets in the air with both hands.* *VCD, IC & TA Move the straight right arm diagonal from up to him until it goes to the back.* *VCD & IC - Opened his right hand with the fingers spread from each other, moved it down.*	*Multiplication* *Diagonal line* *Five*
Iconic Imitate something by physically looking similar.	08:06 -08:07	*A hand with fingers anchor at the table and talk about square root and cube root.*	*VCD -A hand with fingers anchor at the table.*	*Square roots and cube roots*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				

Overlapping gestures These two or three gestures overlap and are not discrete.	08:10- 08:11	*Beating and Iconic gesture opened his right hand with fingers meeting at the tips, and said we have a symbol.*	*VCD and IC - Beating and Iconic gesture opened his right hand with fingers meeting at the tips.*	*Mathematically, he meant symbol or letters*

The analytical tool 2 for the interviews (lesson 4)

Teacher name: Mr. Kuutondokwa (T1) **Topic taught: Power & Base**	**Grade: 8** **Date: 03.03.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
08:06 -08:07	Iconic gesture – VCD	*"I was generally referring to the root. When something is growing, a root is where the body is. A root is a number that make up a square; roots are now referring to natural roots."*
08:07 – 08:08	Metaphor – VCD & TA	*"Just to say what is included inside when you are multiplying".*
08:08 – 08:09	Metaphor – VCD, IC & TA	*That means cutting through. I was showing that the line is not straight but a cut through, i.e. a diagonal line".*
08:10 - 08:11	Overlapping (beating & Iconic) gestures – VCD & IC	*"Just to indicate that it is significant".*
08:12- 08:13	Metaphor – VCD & IC	*"With that I was representing number five".*

The analytical tool 1for the observations (lesson 5)

Teacher name: Mr. Kuutondokwa (T1)		Grade: 8		
Topic taught: Four basic operation		**Date: 4. 03. 2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.				
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	08:08 – 08:09	*Moving open right hand say times zero.*	*VCD, IC & TA – moving open right hand.*	*Zero or 0*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 5)

Teacher name: Mr. Kuutondokwa (T1) **Topic taught: Four basic operation**		**Grade: 8** **Date:4.3.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
8:08-8:09	Beating gesture – VCD, IC & Ta	*“They have to leave out because there is nothing”.*

APPENDIX B: ANALYTICAL TOOLS FOR MS KAMBUTA (T2)

The analytical tool 1for the observations (lesson1)

Teacher name: Ms. Kambuta (T2)		**Grade: 8**		
Topic taught: BODMAS		**Date: 17.03.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	09:27 – 09:28 09:42 – 09:43	*Moved her right hand on both sides of the number in the air next to the chalkboard by making brackets and said "times".* *She made brackets on both sides in the air closer to one learner's book and said use brackets.*	*VCD & JA- Moved her right hand on both sides of the number in the air next to the chalkboard by making brackets.* *VCD, IC & JA - She made brackets on both sides in the air closer to one learner's book.*	*Multiplication* *brackets*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	09:33 – 09:34	*She opened the right hand facing down rhythmically and said "no calculators".*	*IC - She opened the right hand facing down rhythmically.*	*No calculator*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 1)

Teacher name: Ms. Kambuta (T2) **Topic taught: BODMAS**	**Grade: 8** **Date: 17.02.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
09:27 – 09:28	Metaphor gesture – VCD & JA	*"I am telling them to put a division sign"*
09:33 - 09:34	Beating gesture – VCD, IC & JA	*"Put down the calculator".*
09:42 - 09-43	Metaphor gesture – IC	*"I was referring to brackets; putting brackets indicates which one to answer first according to BODMAS"*

The analytical tool 1for the observations (lesson 2)

Teacher name: Ms. Kambuta (T2)		**Grade: 8**		
Topic taught: Division & Multiple factor		**Date: 02.03.2020**		
Five types gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:04 – 10:05	*Open her arm with as if she is hugging and said "whole number".*	*VCD & IC - Open her arm with as if she is hugging.*	*Whole number*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	10:04 – 10:05	*Nodding head and ask learners, how many decimal*	*VCD & TA – nodding head.*	*Decimal place*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 2)

Teacher name: Ms. Kambuta (T2) **Topic taught: Division and Multiplication factors**	**Grade: 8** **Date: 02. 03.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
8:04 – 08:05	Metaphor gesture – VCD & IC	*"This means the whole number"*.
8:04 – 08:05	Beating gesture – VCD & TA	*"I was asking the learners to look at the chalkboard and find the number of decimal places."*

The analytical tool 1for the observations (lesson 3)

Teacher name: Ms. Kambuta		**Grade: 8**		
Topic taught: Director number		**Date: 04.03.2020**		
Five types gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:06 - 10:07 10:24 – 10:26	*She made her body to look like number line by stretching both her right and left arms to the sides to explain positive and negative numbers.* *Moved the right hand up and down and said "above and below sea level" respectively.*	*TA, JA & IC - She made her body to look like number line by stretching both her right and left arms to the sides.* *TA, IC & VCD - Moved the right hand up and down.*	*Mathematically, it means a number line.* *It meant positive and negative numbers on a number line*
Iconic Imitate something by physically looking similar.	10:07 - 10:08 10:32 – 10:34	*Twisted both arms at the knees by bringing hands close to the chest and said less than or greater than.* *She moved her right hand away from the left hand and said, "take out".*	*TA, IC, VCD & JA - twisted both arms at the knees by bringing hands close to the chest.* *TA, VCD & IC - She moved her right hand away from the left hand.*	*less than and greater than* *Minus in mathematics*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				

Overlapping gestures These are two or three gestures that overlap and are not discrete.	10:09 - 10:10 10:27 – 10:29 10:36-10:38	*(pointing and metaphoric) - She moved her left hand far from the head and said number process.* *(Pointing and beating gestures) -Pointed at a drawing of water in the sea on the chalkboard and said, "The drawing is like a number line and where the water end at the drawing it is zero and when going down the water level is negative and when going up the water level is positive number".* *(Pointing and beating gestures) she pointed and beat at a number line while at the same time she was moving to the right side gradually to demonstrate addition.*	*TA, IC & VCD - She moved her left hand far from the head.* *VCD, TA & IC - Pointed at a drawing of water in the sea on the chalkboard.* *TA & IC - She pointed and beat at a number line while at the same time she was moving to the right side gradually.*	*The number line is infinite* *Number line* *Addition on a number line.*

The analytical tool 2 for the interviews (lesson 3)

Teacher name: Ms. Kambuta **Topic taught: Directed number**	**Grade: 8** **Date: 04.03.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
10:06 - 10:07	Metaphor - TA, JA and IC	*"When doing my hand like this, I am representing my head to be 0 and as number goes to the left, the numbers become smaller and as number goes to the right the numbers become bigger. It means the numbers are increasing as you move to the right side and they are decreasing as you move to the left side, from my body."*
10:07 - 10:08	Iconic - TA, IC, VCD & JA	*"Greater than and less than"*
10:09 - 10:10	Overlapping(pointing and metaphoric) - TA, IC & VCD	*"The arrows, meaning the number line continue what limit it is just the space."*
10:24 - 10:26	Metaphor gesture - TA, IC & VCD	*"For the learners to understand that the hip represents the zero. Below it, numbers go down and above, the number go up. For the sea level diagram, when the water rise the numbers are increasing and when the level is dropping, it means the numbers are decreasing. "*
10:27 - 10:29	Pointing and beating gestures - VCD, TA & IC	*"The level of water show the learners that when you are going deep the water you are going below the sea level (i.e. negative numbers) and when you are going above the sea level it represents positive numbers."*
10:32 - 10:34	Iconic - TA, VCD & IC	*"I meant subtraction"*

10:36 - 10:38	Pointing and beating - TA & IC	*"It is like when you have negative numbers it is difficult to understand negative plus positive number but when you have it on a number line. The learners understand that if you are at negative two on the number line, three steps to right will end at negative five."*

The analytical tool 1for the observations (lesson 4)

Teacher name: Ms. Kambuta (T2)		**Grade: 8**		
Topic taught: integers in everyday life		**Date: 5.3.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.	08:15 – 08:16	*She pointed down and said "below water level".*	*JA, IC & VCD - She pointed down.*	*Mathematically, it means negative numbers on a number line.*
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	8:05-8:06	*She moved her left arm vertically with a pointing finger out from other fingers and the right arm moved horizontally with a pointing finger (not with other fingers) and said "positive and negative" respectively.*	*JA & VCD - She moved her left arm vertically with a pointing finger out from other fingers and the right arm moved horizontally with a pointing finger (not with other fingers).*	*Mathematically, she was referring to a number line.*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	08:15 - 08:16	*She moved her right arm up and said, "rise".*	*JA, IC &VCD - She moved her right arm up.*	*refer to "increase" in mathematics*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 4)

Teacher name: Ms. Kambuta (T2) **Topic taught: integers in everyday life**		**Grade: 8** **Date: 5.3.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
08:05 - 08:06	Pointing gesture - JA, IC & VCD	*"It is like when you are at the right-hand side, you point that direction for the learners to know that the numbers on the right hand side also grow bigger. At the negative side, you must point at the left side. The learners will know that a number line has positive and negative numbers."*
08:15 - 08:16	Metaphor gesture - JA & VCD	*"This is above than you go deep, to show the learners that the water level was up then it decreases."*
08:15 - 08:16	Beating gesture - JA, IC & VCD	*"This is above than you go deep, to show the learners that the water level was up then it decreases."*

The analytical tool 1for the observations (lesson 5)

Teacher name: Ms. Kambuta (T2)		Grade: 8		
Topic taught: directed number		**Date: 06.03.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:03 – 10:04 10:03 – 10:04 10:08 - 10:09	*She moved her hand pointing down and said "below sea level".* *She moved her right hand up from the hip and said "above sea level".* *She opened her hand facing down and said "no calculator".*	*VCD - She moved her hand pointing down.* *VCD, TA & JA - She moved her right hand up from the hip.* *VCD & IC - She opened her hand facing down.*	*negative numbers on a vertical number line* *positive numbers on a vertical number line* *no calculator*
Iconic Imitate something by physically looking similar.	10:11 - 10:12 10:21 - 10:23	*She made the + sign with pointing fingers and the small fingers crossing each other.* *She makes parallel lines with pointing fingers and asked learners to draw their private number lines.*	*VCD, IC & TA - She made the + sign with pointing fingers and the small fingers crossing each other.* *VCD - She makes parallel lines with pointing fingers.*	*Addition* *Number lines*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	10:04 - 10:05	*She moved both hands from the chest forward continuously and said, "increase/rise".*	*VCD, IC & TA - She moved both hands from the chest forward continuously.*	*Mathematically, she was referring to increase.*

Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 5)

Teacher name: Ms. Kambuta (T2) **Topic taught: Directed numbers**		**Grade: 8** **Date: 06. 03.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
10:03 – 10:04	metaphor gestures – VCD	*"It means a child must know that with a negative number, you go down, meaning a child should add when it is below or maybe if it was above, the child should subtract drawback from below."*
10:03 – 10:04	Metaphor gestures - VCD, TA & JA	*"It means a child must know that with a negative number, you go down, meaning a child should add when it is below or maybe if it was above, the child should subtract drawback from below."*
10:04 - 10:05	Beating – VCD, IC & TA	*"Increase going up decrease going down"*
10:08 - 10:09	Metaphor gesture- VCD & IC	*"Meaning the learners must not use any calculator".*
10:11 - 10:12	Iconic - VCD, IC & TA	*"No, I was just using my hand I did mean plus, I wanted them to concentrate".*
10:21 - 10:23	Iconic – VCD	*"For the learners to draw their own number line".*

APPENDIX C: ANALYTICAL TOOLS FOR MR. KATAMBA (T3)

The analytical tool 1for the observations (Lesson 1)

Teacher name: Mr. Katamba (T3)		**Grade: 8**		
Topic taught: Irregular polygons		**Date: 15.06.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	08:04 – 08:05	*Moved his right arm side by side as if he is making a cross and asked learners how many 'x' are there.*	*IC & JA - Moved his right arm side by side as if he is making a cross.*	*Mathematically it means x*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson1)

Teacher name: Mr. Katamba (T3) **Topic taught: Irregular polygons**	**Grade: 8** **Date: 15. 06.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
08:04 – 08:05	Metaphor gesture – IC and JA	*"I was just referring the learner's attention to the picture."*

The analytical tool 1for the observations (lesson 2)

Teacher name: Mr. Katamba (T3)		**Grade: 8**		
Topic taught: Angle properties		**Date: 15.6.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.	10:04 - 10:05	*Moved his right arm from one angle to another and said "the opposite angles are equal".*	*JA & IC - Moved his right arm from one angle to another.*	*Mathematically, it meant opposite angles.*
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:08 - 10:09	*Moved his right arm in a clockwise form and told the learners that angles must add up in a clockwise direction.*	*VCD, IC, TA & JA - moved his right arm in a clockwise form and told the learners that angles must add up in a clockwise direction.*	*The mathematical meaning of this gesture is clockwise movement.*
Iconic Imitate something by physically looking similar.	10:05 - 10:06	*Drew a circle in the air around a triangle and said "all the angles inside are equal to 180 ˚."*	*TA, JA & IC - drew a circle in the air around a triangle.*	*he was referring to summing up all angles in a triangle*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 2)

Teacher name: Mr. Katamba (T3) **Topic taught: Angle properties**	**Grade: 8** **Date: 15.6.2020**	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
10:04 - 10:05	Pointing gesture - JA & IC	*"I was referring to opposite angles".*
10:05 - 10:06	Iconic gesture - TA, JA & IC	*"I was just referring to the properties of angles in a triangle".*
10:08 - 10:09	Metaphor gesture - VCD, IC, TA & JA	*"Just telling them to follow one direction, either clockwise or anti-clockwise"*

The analytical tool 1for the observations (lesson 3)

Teacher name: Mr. Katamba (T3)		**Grade: 8**		
Topic taught: Vector and transformations		**Date: 16. 06.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	11:01 - 11:02 11:10 - 11:11	*He moved his pointing finger across and said "horizontal movement".* *He repeatedly closed and opened his hand while saying, "Zero is Zero".*	*JA, TA, IC & VCD - He moved his pointing finger across.* *TA, JA, IC & VCD - He repeatedly closed and opened his hand while saying, "Zero is Zero".*	*Mathematical meaning is x-axis movement.* *It means that when there is zero in a given vector no step must be taken (position remain the same in that direction).*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	11:03 - 11:04	*He shook his hand up and down while holding a mathematical set square. Moreover, he asked the learners a question; "how many steps are we moving?"*	*TA, JA and IC - He shook his hand up and down while holding a mathematical set square.*	*It is showing steps to move in order to locate the next point of vectors.*

Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 3)

Teacher name: Mr. Katamba (T3) **Grade: 8** **Topic taught: Vectors and transformations** **Date:16.03.2020**		
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
11:01 - 11:02	Metaphor gesture - JA, TA, IC & VCD	*"Moved across for learners to differentiate between horizontal and vertical movements"*.
11:03 - 11:04	Beating gesture - JA, TA, IC & VCD	*"I just wanted them to concentrate and be able to see the number of steps moved"*.
11:10 - 11:11	Metaphor gesture - JA, TA, IC & VCD	*"Zero, you have nothing, we don't make any step"*.

The analytical tool 1for the observations (lesson 4)

Teacher name: Mr. Katamba (T3)		**Grade:8**		
Topic taught: Vector and transformations		**Date: 16.06.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.	08:18 - 08:19	*He pointed up and down and said, "We are going to move up and down".*	*JA, TA IC & VCD - He pointed up and down.*	*He was trying to tell learners how to move vectors on the y-axis.*
Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	08:08 - 08:09	*He shook the right arm with its hand open and said, "I only give you time".*	*JA, IC & VCD - He shook the right arm with its hand open.*	*He was referring to time in mathematics*
Iconic Imitate something by physically looking similar.	08:05 - 08:06	*He moved his right arm a distance from the body and asked, "how many meters?"*	*JA, IC & VCD - He moved his right arm a distance from the body.*	*He was referring to length in mathematics.*
Beating Are motions that help emphasize or keep a rhythm in a person's speech.	08:06 - 08:07 08:10 - 08:11	*Moved his right arm far from him with open hands while his fingers were spread from each other and asked, "What is the direction?"* . *Moved his right arm in motion and said, "We are going to move".*	*JA, IC & VCD - Moved his right arm far from him with open hands while his fingers were spread from each other.* *JA & TA - Moved his right arm in motion.*	*This gesture indicated the direction.* *He was referring to the moving of vectors*
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 4)

Teacher name: Mr. Katamba Topic taught: Vectors and transformations	Grade: 8 Date: 16.06.2020	
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
08:05 - 08:06	Iconic gesture - JA, IC & VCD	*"I want to pull their attention to reality of a distance".*
08:06 - 08:07	Beating gesture - JA, IC & VCD	*"I was demonstrating the concept of direction".*
08:08 - 08:09	Metaphor gesture - JA, IC & VCD	*"It was just referring to time".*
08:10 - 08:11	Beating gesture - JA & TA	*"Sign of pulling their attention because in vectors we move. In other words, it is a sign of moving or taking steps"*
08:18 - 08:19	Pointing gesture - JA, TA IC & VCD	*"I was just referring to the movement on the y-axis"*

The analytical tool 1for the observations (lesson 5)

Teacher name: Mr. Katamba (T3)		**Grade: 8**		
Topic taught: Vector		**Date: 18. 06.2020**		
Five types of gestures	**Location in the video recording**	**Description of the gesture and comments. (McNeill, 1992)**	**Functions and actions of the gestures. (Rosborough, 2010)**	**Mathematical Meaning**
Pointing Gestures that point or indicate something.				

Metaphor Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.	10:01 - 10:02 10:01 - 10:02	*Opened both hands down the hip as if he is weighing something or carrying a heavy object. He then asked the learners "is equal to what?"* *Moved his right arm down a bit and said, "This is one over three".*	*VCD, TA & IC - Opened both hands down the hip as if he is weighing something or carrying a heavy object.* *VCD, TA & IC - Moved his right arm down a bit.*	*Mathematically, it means equal.* *mathematical meaning is fraction*
Iconic Imitate something by physically looking similar.				
Beating Are motions that help emphasize or keep a rhythm in a person's speech.				
Overlapping gestures These are two or three gestures that overlap and are not discrete.				

The analytical tool 2 for the interviews (lesson 5)

Teacher name: Mr. Katamba (T3) **Topic taught: Vector**		**Grade: 8** **Date: 18.03.2020**
Location in the video recording	**Types of gestures and functions**	**Transcripts of interviews**
10:01 - 10:02	Metaphor gesture - VCD, TA & IC	*"I am referring to equal."*

10:01 - 10:02	Metaphor gesture - VCD, TA & IC	*"I am referring to fraction."*

APPENDIX D: LETTER OF APPROVAL LETTER, FORM RHODES UNIVERSITY ETHICAL COMMITTEE

RHODES UNIVERSITY
Where leaders learn

Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

7 January 2020

David Haipinge

Review Reference: 2019-0702-3180

Email: g19h2096@campus.ru.ac.za

Dear David Haipinge

Re: Gestures in mathematics education

Principal Investigator: Prof Marc Schafer

Collaborators: Mr David Tuhafeni Haipinge

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.

Sincerely

Prof Joanna Dames

Chair: Human Ethics sub-committee, RUESC- HE

APPENDIX E: CONSENT LETTER TO THE GATEKEEPER

LETTER OF CONSENT FROM THE REGIONAL DIRECTOR

David T Haipinge

P O Box 1884

Oshakati
+264 812 195 673

30 August 2019

The Regional Director of Education, Arts, and Culture

Dear Sir/Madam

Re: Request for permission to conduct educational research at ………….……. School with Grade 8 Mathematics teachers.

I am **David Tuhafeni Haipinge**, a part-time Master student at **Rhodes University** (**student no.: g19h2096**). I hereby humbly request for your permission to conduct my **Mathematics education research study** with three Grade 8 Mathematics teachers at the above-mentioned school for a period of about three to four weeks in the first term of 2020. My study will involve video-recording and interviewing three selected Grade 8 mathematics teachers. The title of my study is: *An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making.*

The study will concentrate on how teachers use gestures as visuals tools to teach mathematics. This will study will help us understand gestures better and to integrate these more strategically into our lesson presentations. It will also help the participant teachers to integrate their gestures more strategically into their own lessons. I intend to observe five lesson per teacher in their natural setting. Stimulus recall interview after each lesson observation will be also conducted with the teachers.

I would like to assure your office that all research ethics principles will be adhered to throughout the process of the study. The identity of the participants and their views will be treated with utmost confidentiality and anonymity. Further, the identity of the schools will not be revealed. I will be using pseudonyms throughout. If desired, a copy of my thesis will be made available to the school. The study is under the supervision of Prof Marc Schafer Email: m.schafer@ru.ac.az). This study has been approved by the Rhodes University of ethics committee director Mr. Siyanda Manqele, email: s.manqele@ru.ac.za.

Thank you for your time, and your consideration and cooperation in this regard will be highly appreciated.
Your cooperation will be highly appreciated.

Yours sincerely

....   ----------------------------

DT Haipinge   Prof Marc Schafer (supervisor)

Student (Rhodes University)

I request you to indicate your choice by ticking [√] in the appropriate box below.

I Agree ☐   do not agree   ☐   that your study be conducted in my region.

Director's name ______________________   Signature: ....

Date:...................................

APPENDIX F: CONSENT LETTER TO THE SCHOOL PRINCIPAL

LETTER OF CONSENT FROM THE PRINCIPAL

David T Haipinge

P O Box 1884

Oshakati

+264 812 195 673

30 August 2019

The Principal

Dear Sir/Madam

Re: Request for permission to conduct educational research at ………….……. School with your Grade 8 Mathematics teachers.

I am **David Tuhafeni Haipinge**, a part-time Master student at **Rhodes University** (**student no.: g19h2096**). I hereby humbly request for permission to conduct my **Mathematics education research study** with three Grade 8 Mathematics teachers at your school for a period of about three to four weeks in the first term of 2020. My study will involve video-record and interviewing selected Grade 8 mathematics teachers. The title of my study is: An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making.

The study will concentrate on how teachers use gestures as visuals tools to teach mathematics. This will study will help us understand gestures better and to integrate these more strategically into our lesson presentations. It will also help the participant teachers to integrate their gestures more strategically in their lessons. I intend to observe five lesson per teacher in their natural setting. Stimulus recall interview after each lesson observation will be conducted with the teachers.

I would like to assure your office that all research ethics principles will be adhered to throughout the process of the study. The identity of participants and their views will be treated with utmost confidentiality and anonymity. The identity of your school will also not be revealed. I will use pseudonyms throughout my thesis. If desired, a copy of my thesis will be made available to the school. The study is under the supervision of Prof Marc Schafer Email: m.schafer@ru.ac.az). This study has been approved by the Rhodes University of ethics committee director Mr. Siyanda Manqele, email: s.manqele@ru.ac.za.

Thank you for your time, and your consideration and cooperation in this regard will be highly appreciated.

Yours sincerely

……………………………… ----------------------------

DT Haipinge Prof Marc Schafer (supervisor)

Student (Rhodes University)

I request you to indicate your choice by ticking [√] in the appropriate box below.

I Agree ☐ do not agree ☐ for my school to participate in this study.

Principal's name ______________________ Signature: ………………………………

Date:…………………………………………..

APPENDIX G: CONSENT LETTER FOR THE PARTICIPANT TEACHERS

LETTER OF CONSENT FOR TEACHERS

David T Haipinge
P O Box 1884
Oshakati
+264 812 195 673
30 August 2019

Dear Sir/Madam

Re: Request to participate in my research study

I am David Tuhafeni Haipinge, a part-time Master student at Rhodes University, South Africa. I am hereby requesting permission from you to be a participant teacher of my research project that I will be conducting with three selected Grade 8 Mathematics teachers for a period of about three to four weeks in the first term of 2020. The title of my study is: *An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making.*

I wish to invite you to be part of this research project by sharing your teaching with me. I am interested in how you gesture in your mathematics class, and how gestures can be used to communicate mathematical ideas. This study will help us understand gestures better and to integrate these more strategically into our lesson presentations. I intend to observe and video record five of your lessons in the first term of 2020 at mutually agreed times. In addition, I wish to interview you on five occasions to assist me in analyzing your video recorded lessons and discuss your perceptions of your use of the gestures you employed.

Your participation in this research study is completely voluntary and you can withdraw at any time you wish. I will ensure that your identity and views will be treated with confidentiality and anonymity. The data that will be collected will not be used for other purposes apart from this study. Findings may be used for conference presentation and articles. It is important that you are aware that this study has been approved by the Rhodes University director of ethical committee Mr. Siyanda Manqele, email: s.manqele@ru.ac.za.

If you have any questions about the research, please feel free to contact me at +264 812 195 673, haipinged@gmail.com, or my supervisor Prof Marc Schafer at m.schafer@ru.ac.za.

Your cooperation will be highly appreciated

Yours sincerely

…………………….

DT Haipinge
Student (Rhodes University)

____

Prof Marc Schafer (Supervisor)

I request you to indicate your choice by ticking [√] in the appropriate box below.

I Agree ☐ do not agree ☐ to participate in this study.

Participant name: ________________________ Signature: ………………………………

Date:………………………………

APPENDIX H: CONSENT LETTER FROM THE PARENTS

LETTER OF CONSENT FROM THE PARENTS

David T Haipinge
P O Box 1884
Oshakati
+264 812 195 673
30 August 2019

Dear Parent

Request for participation in research on Gestures in Mathematics classrooms.

I am David Tuhafeni Haipinge, a full-time Master student at Rhodes University, South Africa. You are being requesting to give permission for your child to be part of the class where I am going to observe how teachers use gestures as visual tools to communicate mathematics meanings. The study will focus on teachers only, not learners. The title of my study is: *An analysis of selected Grade 8 mathematics teachers' use of gestures as visualisation tools to support mathematical meaning-making.*

The study requires me to observe your child's mathematics teacher. I intend to record five lessons for a period of about three to four weeks in the first term of 2020. Kindly be informed that your child's involvement in this study is voluntary. The identity of your child will not be revealed in the video-recordings of his/her teacher as I will record from the back of the class. Please note that you have the right to withdraw your child from the study any time you wish. As the focus of this research study is your child's teacher, no data will be extracted from your child's interactions with his/her teacher.

If you have any question about the research, please feel free to contact me at +264 812 195 673, haipinged@gmail.com, or my supervisor Prof Marc Schafer at m.schafer@ru.ac.za. This study has been approved by the Rhodes University ethics committee director Mr. Siyanda Manqele, email: s.manqele@ru.ac.za.

Yours sincerely

………………………………   ---------------------------

DT Haipinge   Prof Marc Schafer (supervisor)

Student (Rhodes University)

If you agree that your child can participate in this research, please complete the consent form below.

I ………………………………………… (full name of parent/guardian), hereby confirm that

I understand the content of this document and the nature of the research. I am giving permission

to ……………………..………………… (name of the child) to be part of the class where the

study will be conducted

APPENDIX I: ASSENT FORM FOR GRADE 8 LEARNERS

ASSENT FORM FOR GRADE 8 LEARNERS

Dear Grade eight learners

You have had my research project explained to you. You have been given a chance to ask any questions that concern you about my research project. Your parents have given me permission to observe and video record your teacher in your presence. I wish to ask you to agree that I may video record your teacher in your presence. As I will be video recording your teacher from the back of the class your identity will not be revealed at any stage of the recordings.

Nobody will be upset with you if you do not want to be in the classroom during the video-recordings or if you want to stop being in the study. If you have any questions or do not like what is happening, please contact Mr. Haipinge immediately.

By writing your name below, you agree that Mr. David Tuhafeni Haipinge can video-record the lessons of your mathematics teachers.

______________________________ ____________________

Name of a learner (**print**) Date

____

Signature of a learner

_________________________ ____________________

Name of person obtaining assent (**print)**

Date______________________________________

Signature of a person obtaining assent

APPENDIX J: PILOT OF THIS STUDY

Table 2: The analytical tool 1for the observations (with examples from an informal pilot)

<table>
<tr><td colspan="2">Teacher name: Mr/mrs/miss……….</td><td colspan="2">Grade:_________</td><td></td></tr>
<tr><td colspan="2">Topic taught: _______</td><td colspan="2">Date:________</td><td></td></tr>
<tr><td>Five types gestures</td><td>Location in the video recording</td><td>Description of the gesture and comments. (McNeill, 1992)</td><td>Functions and actions of the gestures. (Rosborough, 2010)</td><td>Mathematical Meaning</td></tr>
<tr><td>Pointing
Gestures that point or indicate something.</td><td>For example 11:52-11:53</td><td>Pointing at the board at two points seems like he wants the learners to make a choice between two options.</td><td>JA or SA The teacher is telling learners to substitute in the formulas, numbers at one of the positions on the board.</td><td>Mathematical substitution</td></tr>
<tr><td>Metaphor
Are like metaphors in that they have representative meaning, but do not necessarily look like what they are gesturing about.</td><td>For example 12:07-12:08</td><td>Moving two closed hands away from each other.</td><td>VCD + IC Illustrate to learners that arithmetic progressions have common differences.</td><td>Common difference</td></tr>
<tr><td>Iconic
Imitate something by physically looking similar.</td><td></td><td></td><td></td><td></td></tr>
</table>

Beating Are motions that help emphasize or keep a rhythm in a person's speech.				
Overlapping gestures These are two or three gestures that overlap and are not discrete.	For examples	Pointing and iconic		

Table 3: The analytical tool 2 for the interviews

Teacher name: **Topic taught:**		**Grade:** **Date:**
Location in the video recording	**Types gestures and functions**	**Transcripts of interviews**
8:12-8:18	For example Pointing – JA & VCD	In this block I will paste the excerpts of the interview relating to 8:12-8:18 section of the video recording and add my interpretation of this excerpt in terms of the types and functions of the gestures used.
8:30-8:33		

8:35-8-40		
Etc.....		

APPENDIX K: FEEDBACK FROM A GATEKEEPER

REPUBLIC OF NAMIBIA

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
ASPIRING TO EXCELLENCE IN EDUCATION FOR ALL

Tel: 065 - 229800/25
Fax: 065 - 229834

Private Bag 5518
Oshakati

Enquiries: Hileni M Amukana
Ref. 13/2/9/1

Mr. Haipinge David T
P.O.Box 1884
Oshakati
Cell: 0812195673

SUBJECT: PERMISSION TO CONDUCT A RESEARCH IN OSHANA REGION

Your letter dated 30 August 2019 on the above caption bears reference.

Kindly be informed that permission is hereby granted to conduct research study at Onamuatai SS and Mathews Mundjanima CS in Onamutai Circuit, Oshana Region.

This permission is subject to the following strict conditions; **(i)** There should be minimal or no interruption on normal working schedule **(ii)** Ethical issues of confidentiality and anonymity should be respected and retained throughout this activity i.e. Voluntary participation, and consent from participants and **(iii)** the permission is valid for the academic year 2019/2020.

Both Parties should understand that this permission could be revoked without explanation at any time.

Furthermore, we humbly request you to share your research findings with the Directorate of Education, Arts and Culture, Oshana Region. You may contact Mr. G.S. Ndafenongo, the Deputy Director; Programs and Quality Assurance (PQA) for the provision of summary of your research findings.

We wish you the best in conducting your study.

Yours sincerely,

13/12/2019

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS & CULTURE
13 DEC 2019
OSHAKATI
REPUBLIC OF NAMIBIA

HILENI M. AMUKANA
REGIONAL DIRECTOR

Cc: Inspector of Education: Onamutai Circuit

All Official Correspondence must be addressed to the Regional Director